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A: Agios Loukas, P.C. 65404, Kavala, Greece
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Interest Charges and the “Said” Ageing-related Expenditures: A Study of OECD Countries

Zapji Ymélé Aimé Philombe¹

Department of public finance and taxation, University of Mons/Belgium, <https://orcid.org/0000-0002-1509-4754>

ARTICLE INFO	ABSTRACT
<p>Article History</p> <p>Received 11 January 2023 Accepted 30 January 2023</p> <p><i>JEL Classifications</i> H51, H55, H63, J11</p>	<p>Purpose: The main objective of this paper is to evaluate whether the interest charges on public debt could be a threat for the "said" ageing expenditures. This study attempts to analyze the effects of debt burdens known as interest charges in relation to the pensions and health care spending. The "said" ageing expenditures since the debate on this issue doesn't allow us to say that these expenses are totally linked to ageing.</p> <p>Design/methodology/approach: This study conducts an ordinary least squares analysis based on panel and cross-sectional data covering the period 2000-2020. The data are extracted from OECD statistic and from Eurostat statistic database. The research performs an analysis on 33 OECD countries. The dependents variables are pensions and health care spendings on GDP. The key independent variable is the interest charges. Other additional variables are included in the analysis that we can find in the text.</p> <p>Findings: The results of this study remain ambiguous and call for further study. Nevertheless, based on the current data, there is every reason to believe that, at present, expenditures on interest charges would not crowd out spending on pensions and health care. However, the significance of the demographic variables (old-age dependency ratio, total dependency ratio), and the increase in these ratios in the projections, point to a potential risk of collapse of the pension and health care systems.</p> <p>Research limitations/implications: The main difficulty encountered in this study was the collection of empirical literature dealing with our topic. Many papers used in our empirical literature was not always in relation with the topic of our research. Our challenge was to create the relation with those analyses to propose something original.</p> <p>Originality/value: We propose an innovative study, by proposing the analysis of debt charges in relation to pensions and health care expenditures. Several approaches in the same direction have used other parameters to analyze the costs of ageing, notably the debt to GDP ratio. We integrate other demographic variables such as the dependency ratio, macroeconomic indicators such as the savings rate. All these elements constitute the originality of our study.</p>

Keywords:

Ageing, pensions, health care, interest charges

1. Introduction

The ageing of the population has a growing concern both in the western, and developing countries; its repercussions, similarly, are regarded as of a central debate in recent public policy debates. In fact, the challenges related to the care of the elderly and the generational renewal imply a thorough reflection of the scientific, political and economic worlds. In this context, we are interested in the scientific and economic aspects. The public services associated with ageing are considerable in the advanced countries and involve pension schemes, health care systems, and to some extent education. Their management is primarily a matter for public choice, and finance, as the associated expenditures must provide the most efficient services without compromising current economic outcomes, and the budget sustainability. It is then a matter of resorting to the sustainability of fiscal policy. Moreover, Gruson (2018), emphasizes that fiscal sustainability, in other words the sustainability of fiscal policy, accounts for the state's capacity to ensure the payment

¹ Corresponding author: Zapji Ymélé Aimé Philombe

Email: aimephilombezapjiyme@gmail.com

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of financial charges and to meet the costs of the action programs it has undertaken. Closely related to this, Auerbach et al. (1989), suggest that fiscal policy is efficient if it is able to generate enough revenue to meet or repay the accumulation of debt and associated interests. In the long term, it is a question of looking at the intertemporal budget constraint. Indeed, the challenges faced in relation to ageing are projected over several decades. Experts in the field watch over the data world and European level, or even in Belgium through the Study Committee on Ageing. Thus, in the latest 2021 report of the latter, mandated by the Belgian High Council of Finance, and based on the analyses of Statbel (Directorate-General for Statistics) and the Federal Planning Bureau, it stresses that the costs associated with ageing are increasing as is the elderly population. This conclusion is in accordance with similar observations of European experts and the OECD. If there is one thing that everyone agrees on, it is that age-related expenditures will lead to burden that, if left unchecked, could cause public policy disasters. Experts in the field see several impacts, including fiscal, macroeconomic and social impacts. On the fiscal side, for example, Auerbach et al. (1989), Afflatet (2018), Chen (2004), Ramos-Herrera and Sosvilla-Rivero (2020), Adb Rahman et al. (2021), to name but a few, have shown that fiscal policy will be impacted due to the fiscal imbalance that might be incurred. As pensions are the largest share of the ageing population public expenditures, Grech (2013), Zaidi (2010), as well as many other authors have focused on the sustainability of pensions showing that it is important to pay special attention to reforming the pension systems.

On the other hand, health and long-term care expenditures, although discussed, are also an important part of the burden on public finance. In fact, research by Newhouse (1977), and Dormont et al. (2006), does not show a certain impact of age-related health expenditures on public spending more than other sectors of economic activity. However, Lindgren (2016), and Propper (2005), point out that ageing is also about long-term life support. Since this category of the population very often uses long-term care, it would be wise to carry out an additional study on the additional costs related to long-term care for instance.

In addition, on the macroeconomic level, Disney (1996), Turner et al. (2005), Weil (2006), warn that the decline in productivity linked to ageing would lead to a decline in savings. If this stands true, resources needed for investment would also take a hit. As a trigger, it might compromise economic growth if there should not be any long-term solution found. In fact, economic growth, being the engine from which revenues are generated to finance these costs, could be impacted by this low productivity. States would be faced with an increased debt constraint in order to bridge the income gap.

Debt liabilities imply the repayment of debt and the resulting interest charges. Interest charges are both a cause, and a consequence of public indebtedness. It reflects the burden born in terms of maturing debt repayment, and the associated interest charges, generally expressed as a percentage of GDP. In our previous work, we showed that interest charges, if not controlled, could undermine the sustainability of public finance. In the long run, these charges could prove to be depletive to ageing-related expenditures, hence our study examines whether interest charges can have a crowding out effect related to ageing expenditures. If there is any evidence these days, it is that interest rates on financial markets are relatively low in OECD countries, leading to a reduction in interest charges. This would suggest that the average interest charges in these countries have shrunk in the recent years. We may expect, accordingly, that it provides a relief, or rather a margin for OECD countries to cope with the increased ageing-expenditures projections. However, answering this question requires us to work on OECD countries and by the ordinary least squares method on panel data and on cross sectional data covering the period 2000 to 2020, before giving an answer. Our dependent variables in this study are pensions and health care expenditures as a percentage of GDP. The objective of this paper is then to evaluate whether the interest charges could be a threat for the "said" ageing expenditures. The aim is to propose something innovative by proposing a study that analyses interest charges in relation to the "said" ageing expenditures. Insofar as most studies analyze other fiscal variables such as the public debt.

The rest of the article will be organized as follows: Following this introduction, a second part will focus on the issue of ageing. Starting with a general definition, we will outline the contours of ageing before highlighting its link with fiscal policy. A third part will review the literature on the effects of population ageing. A fourth part will then make an empirical study of our research question. A fifth part will conclude our work.

2. The issue of ageing

2.1 synthetic definition

Although the ageing of the population has become an important issue in debates between politicians and researchers, it is essential to clarify the concept before any analysis. Légaré (2009), informs us that the ageing of individuals is primarily biological and leads to death. It is the process of genetic transformation of man from birth to death. It is ultimately a biological definition and relates to the singular aspect of the individual. On the other hand, when we talk about the ageing of the population, it is strictly structural, the author continues. Several approaches have been proposed to define population ageing, also called demographic ageing. Chen (2004) presents population ageing as a demographic phenomenon that results in an increase in the percentage share of older people in the total population. This tendency is explained by lower mortality rate because of improved well-being, including health care, which accounts for a high proportion of the elderly, and an associated longer life expectancy. The latter is completed by Chesnais (1986), quoted by Légaré (2009), who tells us that it is the decline in fertility that causes the ageing of populations. To understand this phenomenon, we believe that in the early days of technological progress, medical progress first tackled and succeeded in overcoming infant mortality, resulting in younger populations. Then, as mortality, rates were very low, reductions in adult and old age mortality because of improved living conditions and

health care took over gradually led to ageing populations. Demographic ageing is therefore the slowing down of the birth rate and the death rate, which leads to an increase in the proportion of elderly people, or the average or median age of the population. The approaches used to analyze ageing are threefold: population life expectancy, fertility rate and net migration. Population life expectancy simply refers to the average length of human life in a given society. It has increased considerably in recent decades due to advances in medical care that have enabled young children to survive into adulthood. On the other hand, thanks to improved nutrition and other medical advances, more and more people are living to advanced ages (Melyn et al., 2016). The fertility rate reflects the average number of live births in a year to women of childbearing age. For example, between 1946 and 1970, the number of children per woman was 2.7 in OECD countries. Today, the average number of children per woman is estimated at 1.7 in Belgium, for example. Net migration is another important component in changing the age structure of the population. It is defined as the difference between the number of immigrants and the number of emigrants (Melyn et al., 2016).

2.2 Measuring demographic ageing

Studies on demographic ageing have found the dependency ratio to be the determining factor in assessing the impacts of the demographic transition. The old-age dependency ratio, according to the OECD (2018), can be defined as the number of people aged 65 and over per 100 people of working age, i.e., people aged 20-64. This ratio varies over time and among countries. Indeed, in the 1980s, this ratio was 60 years and over for the working population aged 18-60. The low life expectancy at that time could explain this criterion. In addition, in some countries such as Canada, this ratio is analyzed in the 15-64 age group. In fact, the working population ranges from 15 to 20 years old in some countries. However, whatever scale is taken into account, it is that this ratio tends to increase in many countries. The increase or decrease in this ratio, again according to the OECD, depends on mortality and fertility rates and net migration. Between 1946 and 1970, the number of children per woman was 2.7 in OECD countries. Today, this figure is continuously declining. This shows that over time women have on average less children than their mothers, probably due to long studies and career ambitions. On the other hand, the annual net migration according to Melyn et al. (2016), in Belgium, has increased from about 15,000 in the 2000s to 65,000 in the 2008-2011 triennium and this is in line with the OECD countries. There are several reasons why the dependency ratio is important in the analysis of ageing. Nicoletti and Hagemann (1989), insist on this ratio because the fertility rate is particularly unstable. Mortality, on the other hand, is certainly more stable in the medium term, but it too can be difficult to anticipate, given the impossibility of foreseeing any major advances in medicine. Moreover, as changes in lifestyles and health care in the early years of life may affect longevity in ways that remain uncertain, life expectancy may deviate significantly from the figures used in the projections. Immigration, which is strongly affected by political as well as economic factors, is also very difficult to predict, the authors continue. Life expectancy has increased considerably in most rich countries, a trend that most analysts predict will continue, implying an increase in the number of older people and probably in the number of pensioners. This is a wake-up call because its effects are both social and economic. In addition to the ever-increasing life expectancy, the fertility rate has tended to fall, leading to lower figures regarding the entry of the youth into the labor market in the near future. The renewal rate is thus reduced. Some countries that have a favorable trend towards immigration will see this dependency ratio mitigated because the immigrant population will fill the gap of the fertility decline. This is notably the case in Canada, Australia and the USA. In the long term, according to demographic studies, all OECD countries will converge towards an increase in this dependency ratio. Moreover, it can be noted, for example, that in Belgium the dependency ratio has risen sharply since 1950. It was 18.1% in 1950, rose to 25.2 in 1975, 28.3 in 2000 and 30.6 in 2015. It will be 37.1% in 2025, 51% in 2050 and 54% in 2075 if nothing is done. The following table gives an idea of this ratio in some OECD countries in the period 2000 to 2020.

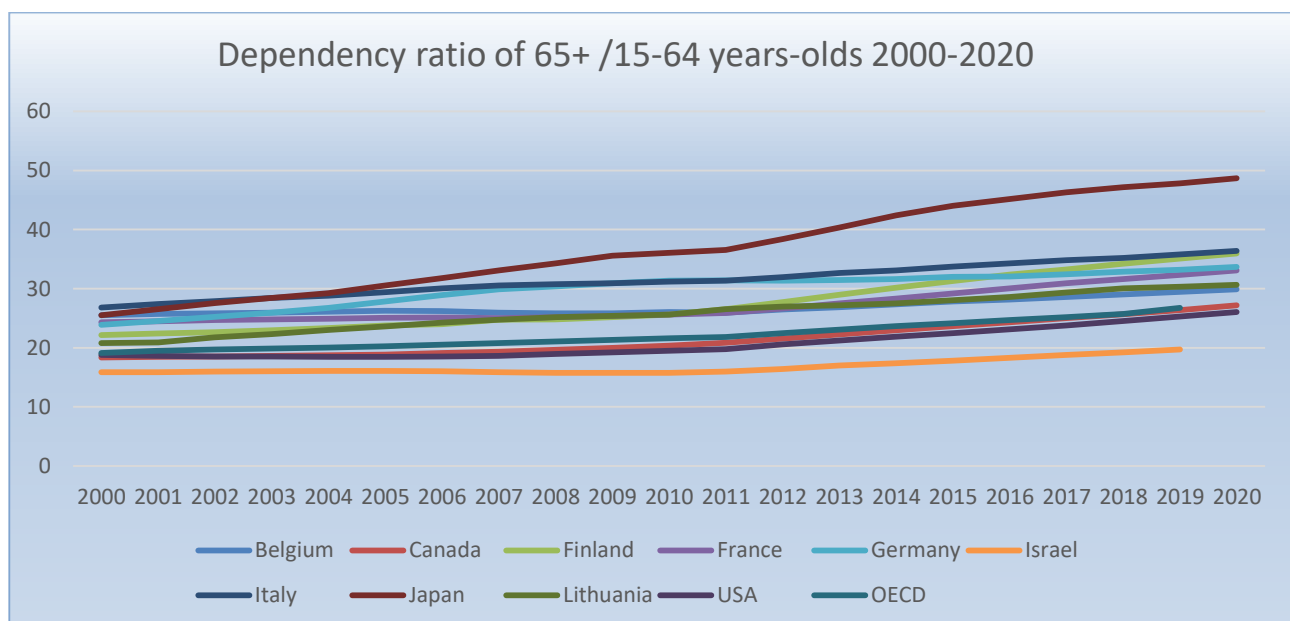


Figure 1: Old age dependency ratio

Sources: Data from OECD statistics, graph by us (Author's construct, 2023).

From the graph above, we can see that there is a tendency for this ratio to increase in the selected sample. It is certainly true that this ratio manifests itself to different degrees, as in the case of Japan, which is already experiencing a concrete manifestation of population ageing. In 2020, this ratio will reach almost 50%. On the other hand, some countries have this ratio fairly contained. This is the case of Israel, which has a ratio of less than 20%. For the rest, this ratio is concentrated between 20 and 30%. Moreover, the OECD average is around 20%.

Although it is recognised that the dependency ratio is unanimously accepted in the scientific world in the study of ageing and its social and economic consequences, other authors have had the merit of providing some criticism. According to Gauthier, H. (1982), in addition to the dependency ratio, other economic criteria should be taken into account, such as participation to the labour market, government spending and all private and future consumption. The idea behind labour market participation is the analysis of the inactivity rate, which determines the share of the population participating in production and income. That is, the more active people there are in a population, the higher the per capita income, taking into account the productivity and employment conditions of the labour force (United Nations, 1969), quoted by Gauthier, H. (1982). Thus, if housewives, the long-term unemployed and all other inactive people are included in the working population, it is clear that there are limits to the dependency ratio. Furthermore, the same author continues, spending on the young and the old are distinct in that, for example, spending on the young is lower than spending on retirement or health care. Despite the limitations that emerge, we believe that the dependency ratio remains the best option for studying population ageing. However, in some works other forms of analysis are proposed, such as the youth dependency ratio, which measures the number of 0-19 years old out of the active population aged 20-64. The total dependency ratio is also proposed. It reflects the total 0-19 and 65+ years of age out of the 20-64 year old working population. Other analysis criteria have been proposed, in particular the dependency ratio of senior citizens, i.e. 80 years and over, which can also be taken into account in the study of ageing. This is because this segment of the population is increasing over time and its impact would appear to be more marked in the evaluation of health care expenditures.

2.3 Demographic ageing and fiscal policy

The analysis of ageing and its relation to fiscal policy is based on five main sectors: Pensions, health care, long-term health care, education and training. In our work we will focus on the analysis of pensions and health care. Pension funding is an important part of public expenditures, accounting for up to a third of total expenditures in many OECD countries. Pensions are most often provided in two forms: pay-as-you-go and funded systems. The pay-as-you-go system has several advantages: it allows pensions from contributors to be paid immediately to beneficiaries; it avoids the risks that inflation might pose to pensioners by linking future pensions to nominal wages. It can provide a higher rate of return to each generation if the sum of the rate of increase in the working population and the rate of increase in wages exceeds the market rate of interest Hageman and Nicoletti (1989). However, it has some drawbacks. It does not in itself increase the volume of resources with which pensions are paid; there is only a transfer of purchasing power from one group (working people) to another (pensioners). Also, it could, if necessary, discourage saving, thus reducing the capital stock from what it would otherwise have been. In the long run, the state will always have a "stock" of uncovered liabilities. This is because the present value of its future pension obligations always exceeds the present value of its future income from existing generations. This leads to debts and deficits. In the funded system, the premiums charged to the insured group (i.e. future pensioners) are set so that the present value of all contributions (past and future) paid by the group is equal to the present value of the future expenses generated by the group. It has the advantage that it can, by inflating the volume of aggregate savings, increase the capital stock and the future level of output. Further, contributions to a funded pension scheme are unlikely to appear to contributors as a tax. The disadvantage usually put forward is that pensions are paid out of the funds and interest paid in, and it becomes difficult to contribute at the full rate as it will take several years. Also, the existence of a very large fund may in itself lead to an increase in the level of government consumption or an increase in the benefits provided to current recipients of public transfers. Many countries are now adopting a bit of both systems to meet pension expenditures.

Health care is also a challenge for public finance. This expenditure can sometimes be equivalent to 10% of GDP in OECD countries and that of long-term care to 1.5%. Indeed, the evolution of the elderly population is often assimilated to an additional cost of health care expenditures, particularly long-term care. However, the debates around this issue tend to favour those who tend to minimise the impact of ageing in relation to care costs (see the work of Hélène Dormont). For the latter and others, the costs of health care evolve with the demand for care, the technology applied and the costs of care personnel. Below we have an idea of the expenditures on health care in a sample of 10 OECD countries.

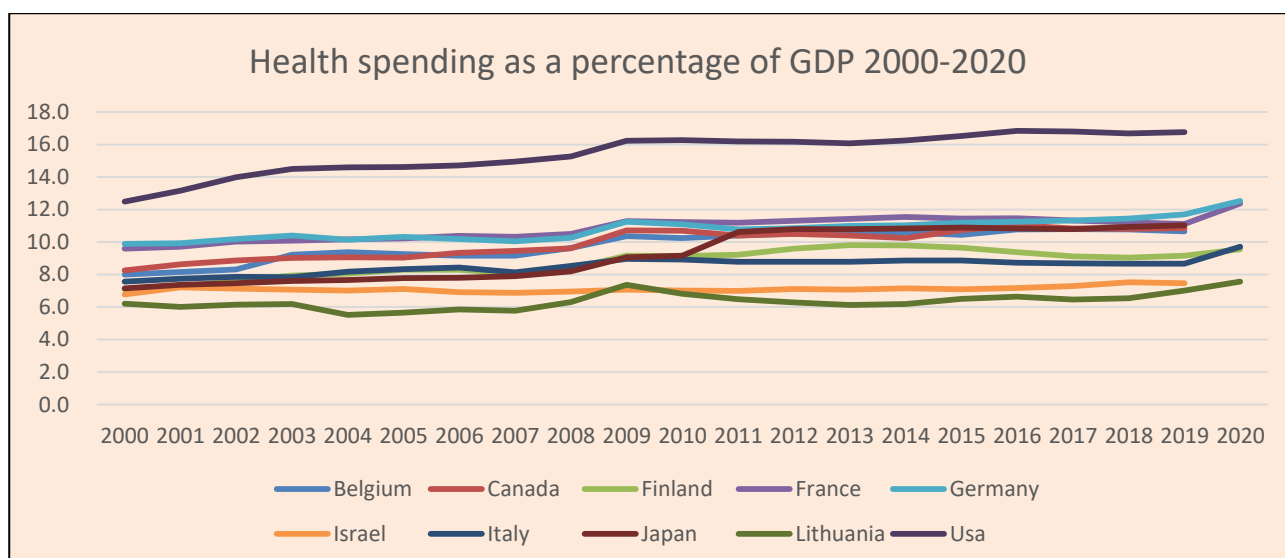


Figure 2: Health care expenditures

Sources: Data from OECD statistics, graph by us (Author's construct, 2023).

The chart above shows that in the OECD countries, at least in our sample, health care spending has tended to stabilise from the 2000s to the present. Only the United States has experienced growth during this period. While in 2000 spending on health care represented 12%, by 2020 it was estimated to be over 16%. This may be due to GDP growth or it may be due to spending on technology and health care personnel. The country with the lowest expenditures on health care is Lithuania, which is around 8%. Countries such as Belgium are in the middle of the two above-mentioned countries and their expenditures has increased slightly since the 2000s to reach 10% of GDP.

3. Literature Review

The analysis of the sustainability of fiscal policy in relation to ageing is a vast field of research for many authors. As we have already stated above, the challenge of population ageing lies in pension benefits, health care and education. Here we will focus on the first two aspects.

The ageing of the population due to an improvement in living conditions brings with it the additional cost of caring for the social categories concerned. The challenge for Western states is to maintain a decent standard of living while keeping budgetary policies viable in the long term. In this respect, Chen (2004), tells us that an unexpected increase in the share of the elderly in the population due to improved longevity tends to increase the budget deficit. This is because it increases the amount of social security benefits that are paid to the elderly by the government. These social benefits, which are financed by taxes and social security contributions, are collected from the working population. If the dependency ratio increases, as is the case in many countries, the situation will be more problematic in the coming decades. Hayati Abd Rahman et al. (2019), make the same point. As the population ages, the number of workers decreases, if there is no change in policies (increase in pension contributions, increase in taxes or reduction in expenditures) this will lead to pressurised public finance. The need for funds will thus force policy-makers to chose indebtment to cover deficits and the associated debt charges. Jensen and Nielsen (1995), confirm this by adding that until the ageing process takes place, if fiscal policy involves a constant debt-to-GDP ratio and balanced budgets, the generations that become old during the ageing period will hardly feel the ageing process. On the other hand, the general tax burden of those who are working at the time the process is underway will be subject to taxes that will increase in line with the increase in old age-related expenditures. Langenus (2006), in order to face both the challenges of ageing and public debt, proposes pre-financing. This would make it possible to relieve current workers and those of successive generations and thus make it possible to achieve the intertemporal budget constraint. The author argues that public finance need to be put on a sounder footing by consolidating them more effectively, which will improve net assets. The creation of net assets will facilitate job creation and productivity, which are the driving force behind the improvement of the primary balance important in pre-financing the costs of ageing. These measures would help to avoid additional pressure, as Hageman and Nicoletti (1989) point out. Excessive pressure encourages the relocation of companies and workers to other countries where the tax pressure is lower. This makes the system in the country of these workers even weaker. For the author, if we have to wait until the moment when public debt accumulates to set up policies, it would be too late and all this would create distortions for those who will be active. If Trandafir Adina (2017), informs us that the sustainability of public finance, which is the capacity of a government to support long-term expenditures without increasing public debt, is threatened, it is nevertheless a question of maintaining the intertemporal budget constraint. For this reason, Cristescu (2019), shows that social sustainability must take into account the risk of poverty of the elderly. This is to remind policy makers that ageing reduces the capacity to work, which in turn reduces income and thus the capacity to save. This will require public finance to find mechanisms to take care of these people. As the dependency ratio is constantly increasing, as the latest publication of the European Union's Ageing Working Group (AWG) reminds us, public deficits and debts and ipso facto interest

charges could increase significantly. Is it not therefore fundamental to take into consideration the sustainability of pension systems?

The analysis of pension systems is a crucial aspect in the analysis of ageing as it entails huge burden and therefore needs to be handled with care. Schneider Cesifo (2009), speaking about pension reforms in Europe, considers that a pension reform is considered successful if it reduces future expenditures. In other words, the sustainability of a pension system lies in its capacity to reduce benefits, especially pensions. This view that pensions are expensive for public finance and should in principle be reduced to avoid excessive burden on future generations is challenged by Grech (2013). For him, pension plans should integrate three factors, namely, public expenditures, pensioner poverty and the level of income replacement. Therefore, cutting expenditures should not be considered as the right answer to ageing. A system should be able to guarantee benefits to the whole population to prevent old age poverty and offer ways to smooth consumption over the life cycle. This means analysing situations on a case-by-case basis. Indeed, as already suggested by the OECD (2005), quoted by Grech (2013), a country with a lower life expectancy could afford to pay higher replacement rates to its citizens while imposing the same contribution on its workers as a country with a higher life expectancy. This is in line with the idea of the public support ratio analysed by Lee and Mason (2016). Where they show that the analysis of expenditures that integrate the pension system should include the public support ratio. If the public support ratio is around 1, the primary budget is balanced and no sphere of society (workers and pensioners) should be under pressure. However, if the ratio is below 1, benefits may have to be reduced or taxes increased. Similarly, if this ratio is higher than 1, it means that benefits can be increased or taxes can be reduced without affecting the budget balance. The answer to the question of the future of pensions in the face of ever-decreasing fertility rates has been a leitmotif for some authors. In this context, the work of Oksanen (2009; 2016), has made it possible to examine the notion of generational equity in the design of pension reforms. To this end, he assumes that if the incoming and outgoing generations are balanced, i.e. the young contributing for the old, there would be no worries. However, if the current generations have a lower birth rate, it makes sense to increase pension contributions, thus increasing savings and capital, or to raise the retirement age in order to avoid a double burden on future generations. Should we not assess whether immigration could be a solution to the sustainability of public pensions as proposed by Serrano et al. (2011). Godbout et al. (2010), argue in the context of intergenerational equity to calculate in the projections, as was the case in Canada, the volumes of taxes and tariffs that will be required, uniform and constant from the year of projection until the next 50 years. The income earned would be set aside for a certain period of time and then later spent on new generations. Actuarial neutrality would then mean that the degree of pension funding would change in line with fertility, longevity and pension policy parameters. This implies that adults have to compensate for lower fertility and increased longevity by increasing their retirement age. A problem of equity could still arise. For this reason, Oksanen (2016), reminds us that while it is verified that the post-World War II generations have not contributed much to personal pensions, they have nevertheless contributed to the accumulation of savings and capital. And, taking into account the multiplier effects of economic growth, would it not be interesting to take this into account when evaluating pension reforms, as many experts recommend in Western countries. The replacement rate should therefore take this factor into account. Other recommendations for improving pension systems are offered by Van Meensel et al. (2016), Bazzana (2020), Van der Horst et al. (2011), Hageman and Nicoletti (1989) and D'Autume (2003), who emphasise reforms of both the pay-as-you-go system, which most OECD countries use, and the funded system. What emerges is a gradual increase in retirement ages. It would improve fiscal sustainability. Indeed, the extension of the active period is synonymous with additional budgetary revenues and less expenditures. Bazzana (2020), for example, teaches us that when the share of workers is higher than the share of retirees, the pension system is fully sustainable. Because the tax base is broadened, total output itself improves and thus resources are available to improve the state budget. This somehow helps to meet the challenges of ageing and to solve the problem of interest charges on the public debt. Also, the extension of the retirement age allows the distribution of the social burden among all social components. Indeed, when these burden rest on a few workers, the tax pressure on these workers is enormous, which will tend to discourage some from working, and since we know that a worker receives few transfers from the state (unemployment, health care, social security), it is preferable to encourage a large number to work and to stay at it as long as possible. In OECD countries where there is a slowdown in fertility, an increase in retirement ages would only be beneficial, as it would help to keep the demographic dependency ratio at sustainable levels. Also, reforms of the pension system should encourage greater participation of women in the labour market. Greater participation of women in the labour market reduces the number of dependents on the state, increases tax contributions and decreases dependency ratios. Hageman and Nicoletti (1989), would like the authorities to review pension replacement rates. Indeed, to receive a full pension, most systems do so on the basis of final gross earnings. Our authors propose, for example, to use net wages as a basis, which would make the pension system less generous.

Health care is a determining factor for social well-being. As such, it is one of the determinants of improved living conditions and is strongly associated with increased life expectancy. However, over time health care costs become considerable, and are rightly or wrongly attributable to advanced ages. In the literature reviewed, the works of Dormont (2010 and 2012), Dormont and Huber (2009), Dormont et al. (2006), are firm on the argument generally put forward concerning health care costs. For these authors, the increase in health care costs would not find its major explanation in ageing. It is true that individuals see their health expenditures increase with age, but the primary explanation is that as time goes by, individuals spend more on their health. In 1990, for example, an individual suffering from cancer would spend less than in 2020 for the same disease. Also, as people age they are more likely to develop certain diseases. Here it is primarily a question of biology. Thus, a person whose age changes should still have health concerns, and this would not be an additional cost to state health care spendings. Furthermore, the type of

disease explains the cost of expenditures independently of age. Indeed, people with chronic diseases (e.g. renal failure) need periodic treatment, and incur recurrent intervention financed through the healthcare system, and even if they are insured, the public intervention is more considerable given the technologies used in the case of renal failure for example. Co-morbidity factors are also another reason for soaring health costs. Although the proportion of the population mainly subject to co-morbidities (a set of pathologies) is found among the elderly, other social categories are also affected. As a result, people with several types of illnesses will naturally be less productive and will therefore have to be supported by the social system and this will further increase the costs of care. Tenand (2014), follows the same line as the previous authors, but adds the types of care as another reason for the costs. Indeed, once a disease has been diagnosed, the costs will depend on the treatments available and used, as well as the medical interventions that follow, including consultations, examinations and the type of clinic, to name but a few. The costs of care are to be found in the years before death. The previous authors have shown that the closer one gets to death, the greater the costs become. If we refer to palliative care units, it is clear that these patients are being cared for with enormous resources, including drugs, real estate and the cost of nursing staff. Technological progress necessarily plays a major role in increasing or reducing these types of costs. Newhouse (1977), already argued that health technologies play a minor role in increasing health care costs and his ideas have been the subject of other studies, such as Westerhout (2006), who highlighting the elements presented above, analysed the role of technological development in health care. It is clear that while technological development lower costs in some sectors, it does the opposite in the health sector in different situations. Dormont (2009), mitigates this by pointing out that the diffusion effect (the nature of the innovation of new treatments to a greater or lesser number of patients) leads to an increase in expenditures, while the substitution effect (i.e. new treatments replacing others) leads to a decrease in costs. However, the explanation for the growth in the costs of technological innovation is largely due to patents, the costs of training health professionals and of course public policies in this area.

At the macroeconomic level, the effects of ageing are manifested in particular on capital and savings. Dave Turner et al. (1998), report that in OECD countries, the decline in savings in the future seems to be confirmed. Whether it is private or national savings, this reduction will put upward pressure on real interest rates, which in turn will lead to a slowdown in capital accumulation. Being strongly linked to capital, investment would be reduced and this will influence the growth of the economy and therefore the overall wealth. To understand the macroeconomic impact, we start from the analysis of the life cycle of the saver proposed by Disney (1996). Indeed, the slowdown in savings linked to ageing stems from the fact that in the human life cycle, savings are high when people are working and they are therefore net savers at that time. As he ages, he gradually exits the labor market and becomes a dissaver. In macroeconomic terms, if the dependency ratio continues to be persistently high, it is easy to understand that the household savings rate will be lower. As Weil (2006) argues, capital accumulation allows individuals or society as a whole to break the temporal link between production and consumption, i.e. an individual, for example, can save part of his or her wage while working and then use the accumulated capital to finance consumption during retirement. Recent state reforms to raise the retirement age in line with increasing life expectancy serve to increase aggregate savings, reduce social security contributions and increase the tax base. Afflatet (2018), on the other hand, points out that the increase in the old-age dependency ratio leads to a decrease in the labor force which, in turn, should lead to a decrease in the growth rate. In the same sphere, he states with regard to interest rates, that it could be expected that older people entering retirement would deplete their savings, leading to a higher interest rate due to a reduced supply of capital. Baumol (1967), will propose an analysis of productivity by stating that the steady growth of productivity is the result of technological innovation manifested by new capital goods. Capital goods are thus the source of economies of scale, which is another source of productivity growth.

4. Empirical analysis

4.1 Synthetic empirical literature review

Population ageing has or will have effects on the sustainability of public finance. To analyse these effects from a scientific point of view, we need to look at some of the articles that will inspire our work. Here we propose a synthetic review which is however far from being exhaustive.

Ramos-Herrera and Sosvilla-Rivero (2020), analyse the effects of ageing on the sustainability of fiscal policy in 11 European countries covering the period 1980-2019, while controlling for macroeconomic variables (real economic growth, financial development, the inflation rate, the trade balance, the effective exchange rate, the output gap and, of course, demographic ageing). Working on panel data from Eurostat, the results of this study show that ageing has generated profound pressures on fiscal sustainability. With a negative and statistically significant coefficient, the budget balance deteriorates on average by about 21.30 percentage points for each percentage point increase in the old-age dependency ratio. Fiscal policy has therefore not been compatible with long-term sustainability.

Afflatet (2018), analyses the impact of population ageing on public debt. He works with ordinary least squares on panel data for 18 European countries covering the period 1980-2015, and these data are taken from Eurostat. In his analysis, it is shown that there is little empirical evidence of such an impact until 2015. He works with different indicators to capture the ageing of the population such as the total dependency ratio (0-15 and 65+/15-64), the old-age dependency ratio (65+/15-64), the over 85 dependency ratio (85+/15-64). It also inserts other macroeconomic variables such as per capita income, unemployment, investment and growth. The results for the demographic variables show that the total dependency ratio and the old-age dependency ratio are significant in reducing the debt. By splitting the dependency ratios of the under 85s and the over 85s, this result does not fundamentally change. The same is true for the dependency ratio of the under-15s.

Abd Rahman et al. (2020), analyse the external debt of states in relation to ageing. Other control variables are included in the model such as inflation, GDP growth, interest rates, gross debt. The latter use a panel of 36 countries between emerging and developing countries over the period 2000 to 2017. The data is taken from the World Bank's World Development Indicator and the International Monetary Fund's World Economic Outlook. They use the generalized method of moments (GMM), which shows that population ageing has generated a significantly positive relationship with the external debt of the IMU, but with a very small impact (between 0.058% and 0.063%) when the independent variable is the percentage of the population aged over 65. In contrast, the old-age dependency ratio did not show a significant correlation.

Cristescu (2019), using Eurostat data and an econometric study on panel data, analyses the effects of the risk of poverty among the elderly using ordinary least squares. She observes that some European countries will experience a demographic decline while others will be clearly spared. However, the cost to public finance will remain considerable whatever the scenario. Analysing the 28 European countries over the period 2005-2017, the sustainability of pensions is at the heart of his study. The results show that life expectancy is negatively correlated with the risk of poverty in old age. Also, the dependency ratio is positively correlated with the risk of poverty. Because the higher the dependency ratio, due to a reduction in the active population and thus a low entry of young people into the labour market, this puts additional pressure on fiscal policy. The same situation is true for the increase in health care costs. In fact, an increase in health care costs crowds out other expenditures such as pensions, which leads to an increase in the risk of poverty among the elderly.

Schneider (2009), analyses pension reforms in 17 member countries of both the OECD and the European Union based on panel data. He performs an ordinary least squares econometric analysis of the index of pension reforms on variables such as: trade union power, fiscal institutions, general government expenditures, pension expenditures, pre-funding of pensions, public debt and the demographic dependency ratio. The results obtained show a weak correlation between the demographic dependency ratio (significant at 10%) and pension reforms. The collective bargaining power of workers and current pension expenditures are largely significant (between 5 and 1%). Other variables such as public debt, the ratio of workers with pension funds and the change in pension expenditures are not significant.

Razin et al (2002), analyse population ageing and the size of the welfare state. The regressions elements are: the labour tax rate and real transfers per capita on the population dependency ratio. Additional control variables are introduced such as public employment, GDP growth and the degree of openness. Most of the data are taken from OECD statistics. The basic idea of the analysis is that states with a high tax rate are those states with the highest public transfers. Using an ordinary least squares econometric analysis in panel data, the authors conclude that the dependency ratio has a statistically significant negative effect on the labour tax rate, which removes the ambiguity of the analytical model. A one percentage point increase in the dependency ratio leads to an almost 0.4 percentage point decrease in the labour tax rate. A higher dependency ratio leads to lower transfers per capita, the coefficient being statistically significant in all specifications.

Chen (2016), conducted an ordinary least squares econometric analysis, where he deals with the effects of the age structure of the population and the budget deficit by analysing panel data with fixed effects. The data are estimated over the period 1975-1992. He analyses the population structure on the primary surplus by also including a lagged dependent variable to reduce the serial correlation of the error terms. The regression results indicate that an increase in the shares of the young population and the elderly tends to decrease the budget surplus shares only in developing countries. For developed countries, the author indicates that increases in the shares of the young and elderly population tend to increase the share of the budget surplus in GDP, with the estimated coefficient for the elderly population share being significant. Other variables such as the working population are naturally significant. Indeed, the more the population works, the more the state has the means to ensure its regalian missions.

Jochen Hartwig (2008), taking up Baumol's (1967) theory on the determinants of health expenditures, whose conclusions showed that health expenditures are determined by higher wage increases linked to productivity growth, which leads to a directly proportional growth of health expenditures. The author conducts an empirical study using ordinary least squares in cross-sectional and panel data on 19 OECD countries covering the period 1960-2004 and from 1990 to 2003 for France. The data are taken from the health section of OECD statistics. The variables analysed in his model include current expenditures on health care as the dependent variable, labour productivity, wage growth and real GDP growth as independent variables. The regression results indicate positive significance at the 1% level for labour productivity growth, real GDP growth and wage growth.

4.2 Selection of variables and models

As mentioned above, our study focuses on debt servicing in relation to the costs of ageing. The question is to see to what extent public debt charges are likely to crowd out the costs associated with population ageing. The literature analysed above has highlighted several important elements that could explain our problem. We will draw inspiration from them for the choice of our variables, for which there are no typical cases in the literature.

As mentioned above, our work focuses on the analysis of pensions and health care costs in relation to debt charges. We propose here the variables to be used in the first model. Naturally, our first dependent variable is the pension as a percentage of GDP (PENSION). In OECD countries, public pensions represent more than 10% of GDP, especially in Western Europe, as in Belgium, where they are expected to rise to 13% by 2021. Several authors have worked on the subject, such as Ondrej Schneider (2009), and Cristescu (2019), and have shown that they have a considerable impact on fiscal policy. Our key variable is the burden on public debt as a percentage of GDP (DEBTCHARGE). It is closely related to public debts and therefore should have a considerable impact on social spending. We naturally integrate the demographic variables (DEM), the first element of which is the old-age dependency ratio (65+/15-64 years) defined

above. Subsequently, we will integrate the total dependency ratio (0-15 and 65+/15-64 years), the youth dependency ratio (0-15/15-64 years), the 65+ and 80+ populations into the total population. Other control variables are also inserted in our model. Thus, General Expenditures as a percentage of GDP (GENEXP) is associated with it. In the analysis of the sustainability of public finance, it is interesting to integrate the overall government expenditures, as it is expected to explain or at least have an impact on pension expenditures. The variable GDP growth (GDPGROWTH) is also inserted in our model. Working in the macroeconomic sphere, it is interesting to insert it because we believe that it would explain our dependent variable. Our model also incorporates savings over GDP (SAVING). Savings or capital is decisive in studies of this type as Disney (2016), alluded to. We have seen in the literature that it determines the potential capital to be invested and we believe that in pensions it would have an influence. In fine, we will insert the variable taxes (TAXES). In fact, in sustainability analysis, taxation is always put forward as a determinant of public expenditures because the financing of the latter depends on the former. Other variables will be inserted later in order to test the robustness of our model. Our model is presented as follows:

$$PENSION(i,t) = \beta_1 DebtCharge(i,t) + \beta_2 DEM(Rate\ 65+/15-64)(i,t) + \beta_3 GdpGrowth(i,t) + \beta_4 GenExp(i,t) + \beta_5 Saving(i,t) + \beta_6 Taxes(i,t) + \mu(i,t). \quad (1)$$

As our work also focuses on the analysis of health expenditures in relation to public debt charges, we propose to distinguish two models to avoid misinterpretation. Health expenditures as a percentage of GDP (HEALTH) is our second dependent variable. Our key variable is public debt charges as a percentage of GDP (DEBTCHARGE). The choice of this variable is in line with the problematic posed above. Chen (2016), for instance, in his study uses primary surpluses to analyze sustainability, while Schneider (2009), uses public debt, to name but a few. We naturally integrate the demographic variables (DEM), the first element of which is the old-age dependency ratio (65+ / 15-64 years) defined above. Subsequently, we will integrate the total dependency ratio (0-15 and 65+ / 15-64 years), the youth dependency ratio (0-15 / 15-64 years), the 65+ and 80+ populations into the total population. Other control variables are inserted in our model, notably GDP growth (GDPGROWTH). Here we are inspired by Hartwig (2008), Herwartz and Theilen (2003), who show that health care expenditures cannot be explained by ignoring GDP growth. Previous studies, including Newhouse (1977, 1987, 1992, 2001), have shown that much of the spending on health comes from the fact that once a state's GDP increases, welfare is improved by spending more on health care, training, increased wages and improved quality of life. We have included General Expenditures as a percentage of GDP (GENEXP) as another control variable. Our model also integrates savings over GDP (SAVING), so we want to integrate economic variables to analyse their impact in reducing or increasing health care costs. Subsequently, we included other variables to test the robustness of our model. Labour productivity (PROD) is one of our control variables. We included it because we believe, like Hartwig (2008), that good productivity in the care sector improves the quality of work and reduces costs. We include the activity rate of 65-69 year olds (ACTIVERATE 65-69) in order to determine whether this category of active people would have an impact on health care expenditures. We have also included a technology variable (TECHMED) in our regression. This is in line with what Tenand (2014), following Dormont et al. (2009), described as the non-demographic determinants of health care expenditures. Here we have chosen to insert the medical technology of "CT scans", which are measured in terms of the number of scans per million inhabitants. Our model gives this:

$$HEALTH(i,t) = \beta_1 DebtCharge(i,t) + \beta_2 DEM(Rate\ 65+/15-64) + \beta_3 GdpGrowth(i,t) + \beta_4 GenExp(i,t) + \beta_5 Saving(i,t) + \mu(i,t). \quad (2)$$

Our sample covers the period 2000 to 2020. Our data are extracted from OECD statistics for most. The pension data for European countries are taken from Eurostat. We work on 33 of the 37 countries that make up the OECD. The following countries: Chile, Colombia, Mexico and Turkey have not been included in our study, as data from these countries are not always available or irregular. Our analysis will be carried out by the ordinary least squares method and our data are in panel. Working on a time arc and on several countries, it is thus obvious that panel analysis is the appropriate method. As our data are not perfect, we will have to conduct basic tests to avoid endogeneity, multicollinearities or even heteroscedasticity. The Hausman test allowed us to choose in both models a fixed effect regression that we regress with standard robustness of errors. The multicollinearity tests allowed us to evacuate in our regressions the highly correlated independent variables. The Breusch-Pagan Heteroscedasticity tests were conducted in our regressions. In addition, to correct for potential endogeneity errors we introduced in the regression the variable lagged by one period the dependent variable in model 4. We also lagged the variables DEBTCHARGE, GDPGROWTH, GENEXP, SAVING by one period. Our models take the following form:

$$1- PENSION(i,t) = \beta_1 DebtCharge(t-1)(i,t) + \beta_2 DEM(Rate\ 65+/15-64)(i,t) + \beta_3 GdpGrowth(t-1)(i,t) + \beta_4 GenExp(t-1)(i,t) + \beta_5 Saving(t-1)(i,t) + \beta_6 Taxes(t-1)(i,t) + \mu(i,t). \quad (3)$$

$$2- \underset{(4)}{HEALTH(i,t)} = \underset{+}{\beta_1 Health(t-1)(i,t)} + \underset{+}{\beta_2 DebtCharge(t-1)(i,t)} + \underset{+}{\beta_3 DEM(Rate\ 65+/15-64)(i,t)} + \underset{+}{\beta_4 GdpGrowth(t-1)(i,t)} + \underset{+}{\beta_5 GENE xp(t-1)(i,t)} + \underset{+}{\beta_6 Saving(t-1)(i,t)} + \underset{+}{\mu(i,t)}.$$

The following table describes the variables used in our regressions and the sources where there have been extracted.

Table 1: Variable description

VARIABLE	DESCRIPTION	DATA SOURCES
Pension	Pension expenditure on GDP.	Eurostat & OECD
Health	Health care Expenditure on GDP, not including long-term care.	OECD
DebtCharge	Debt burden on GDP.	OECD
GDPGrowth	GDP growth aggregate per year.	OECD
GenExp	Total government expenditure as a percentage of GDP.	OECD
Saving	Aggregate savings as a percentage of GDP.	OECD
Taxes	Average income tax (% gross wage earnings). Single person at 100% of average earnings, no child.	OECD
ActivityRate 65-69	Labor force participation rate of people aged 65-69.	OECD
LaborProductivity	Labor productivity is defined here as output volume divided by total input labor.	OECD
MedicalTech	Tomography scanner used in medical imaging to scan the human body.	OECD
DepRate 15-64Y	The old-age dependency ratio is the number of people aged 65 and over per 100 people of working age, i.e. people aged 20-64.	OECD
DepRate 0-15Y	The youth dependency ratio is the number of people aged 0-15 per 100 people of working age, i.e. people aged 20-64.	OECD
DepRate 0-15 & 65Y+	The total dependency ratio is the number of people aged 0-15 & 65 and over per 100 people of working age, i.e. people aged 20-64.	OECD
Pop 65Y+	Population aged 65 and over out of total population.	OECD
Pop 80Y+	Population aged 80 and over out of total population.	OECD

Source: (Author's construct, 2023)

The descriptive statistics of our study are presented in the table below.

Table 2: descriptive statistic

	Obs	mean	Var	St.Dev	min	max
Pension	651	9.215295	13.65563	3.695352	1.078	17.9
Health	680	8.58649	4.335326	2.082144	3.898	16.844
DebtCharge	671	1.507228	2.614434	1.616921	-3.180441	7.269771
GdpGrowth	693	4.307808	24.49808	4.949553	-22.60013	34.75724

GenExp	693	43.67434	50.67845	7.11888	23.68063	65.10932
SavingGDP	662	6.736831	34.49241	5.873024	-12.8465	27.50576
Taxes	693	16.73069	44.01295	6.634225	2.205467	38.5842
ActivityRate65-69	685	17.73032	156.7537	12.52013	1.078685	55.56751
LaborProductivity	693	0.9494023	0.0088677	.0941687	60.5195316	1.162237
MedicalTech	511	22.98472	197.1911	14.04247	4.42	111.49
DepRate 15-64Y	692	24.43898	29.01688	5.386732	10.07	48.71
DepRate 0-15Y	690	25.7327	24.92823	4.992818	17.63	47.17
DepRate 0-15 & 65Y+	690	50.16913	28.99044	5.384277	36.19	68.85
POP 65Y+	693	16.20058	10.41092	3.226596	7.2	28.9
POP 80Y+	693	4.116883	1.431839	1.196595	1	9.3

Data are from OECD statistic and Eurostat, Table made with Stata.

4.3 Interpretation and results

The results of our various regressions are reported in the tables below:

Table 3: regression on pension							
Dependent var: Pension	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5	Reg. 6	Reg. 7
LagDebtCharge	-0.051 (0.096)	-0.105 (0.099)	-0.146 (0.114)	-0.050 (0.095)	-0.061 (0.106)	-0.045 (0.110)	-0.156 (0.102)
DEM(Rate 65+/15-64)	0.124*** (0.025)						
LagGdpGrowth	-0.041*** (0.014)	-0.050*** (0.016)	-0.046*** (0.016)	-0.038*** (0.013)	-0.035*** (0.013)	-0.053*** (0.016)	-0.039*** (0.014)
LagGenExp	0.088*** (0.017)	0.096*** (0.020)	0.099*** (0.026)	0.087*** (0.017)	0.088*** (0.020)	0.095*** (0.022)	0.102*** (0.019)
LagSaving	-0.067** (0.025)	-0.055** (0.025)	-0.065** (0.031)	-0.070*** (0.025)	-0.062** (0.029)	-0.056** (0.027)	-0.076*** (0.025)
LagTaxes	0.049* (0.027)	0.047 (0.033)	0.048 (0.039)	0.049* (0.027)	0.062* (0.031)	0.077** (0.036)	0.064* (0.037)
DEM (Rate 0-15 & 65+/15-64)		0.104*** (0.025)					
DEM (Rate 0-15 / 15-64)			-0.069 (0.058)				
DEM (Pop 65 ans+)				0.224*** (0.046)			
DEM (Pop 80 ans+)					0.499*** (0.126)		
ActivityRate 65-69						0.056*** (0.017)	
LabProductivity							2.469* (1.237)

Constant	2.548*	0.076	7.010***	1.990	3.242**	3.867***	2.575
	(1.284)	(1.908)	(2.039)	(1.355)	(1.323)	(1.243)	(2.033)
Observations	576	576	576	576	576	573	576
R-Sq	0.544	0.507	0.417	0.544	0.538	0.452	0.446
N	32	32	32	32	32	32	32

Robust Standard Errors in brackets. *** p<0.01, ** p<0.05, * p<0.1

The regressions reported in Table 3 provide results that we will attempt to analyze and interpret. In the first regression, our key variable, the Lag of debt charges (DEBTCHARGE), did not prove to be significant, but the negative sign tells us that if it were to be significant, it would contribute to lowering pension expenditures. This result is in line with the study of Schneider (2009), who showed that public debt was not significant. Since debt charges are closely related to public debt, we retain this result. This regression answers our question of whether debt charges would crowd out ageing expenditures. Since our key variable does not explain pensions, it would not be a threat to the sustainability of pension expenditures. As we have shown in many OECD countries this ratio is sometimes less than 1% of GDP. Our second variable DEM (Rate 65+/15-64) is unsurprisingly significant and positively (1%) correlated with pension expenditures. A one-percentage point increase in the old-age dependency ratio leads to a 25% increase in pension expenditures. Schneider (2009) has done a similar study and shows that the dependency ratio is positively correlated with pension reforms. This shows why it is important for policy makers to pay close attention to this ratio. Because if it is increasing at a rapid pace and there are not enough workers entering the labour market, especially in pay-as-you-go pension systems, fiscal policy will require either cost-cutting in other sectors, increased taxation, cuts in general expenditures or reforms such as raising the retirement age. Other control variables inserted in the analysis include Lag of GDP Growth (GDPGROWTH), which in our regression proved to be negatively significant. Razin et al. (2002), who analyzed the growth of GDP per capita over social transfers, obtained a similar result. This result seems strange to us, as one would have expected the sign to be positive. This is because the more the economy grows, the more possibility for manoeuvre there is for the government to take care of people in retirement. There is a consistent explanation for this result. In fact, when the economy grows, this is reflected on citizens via wages and transfers. In this case, purchasing power increases, which gives citizens the possibility to subscribe to private or supplementary forms of pension, thus, reducing the governmental burden. In addition, the regression of the LagGenExp variable (GENEXP) obtained a significant and positive result (1%) with pensions; which is not in itself a surprise. Because when the public budget foresees an increase in overall expenditures, this translates into an increase in transfers to citizens, including the elderly. The variable LagSavingGdp (SAVING) was found to be negatively significant at the 5% level. This is explained by the fact that the accumulation of savings reflects an increase in wealth and capital, which means an increase in wealth per capita. Citizens will be inclined to subscribe to other forms of pensions (supplementary and private) thus reducing government expenditures. Turner et al. (1998), Disney (1996) and Weil (2006), have drawn attention to the importance of savings because if savings decline, investment will decline, aggregate and pro-capita GDP growth will fall significantly. Interest rates at that time will rise. The variable LagTaxes (TAXES) is weakly significant at 10% of the confidentiality threshold. The sign is in line with what we expected. In fact, the increase in taxes reflects a restrictive fiscal policy to correct the government's financing needs. In the case of an increase in the old-age dependency ratio or of people going into retirement, the increase in taxes translates into an increase in pension expenditures. As regression 1 was our base model, other variables were included in our work as controls. In regression 2 we have included the variable DEM (total dependence rate (0-15 and 65+/15-64 years)). The sign is positive as in regression 1 with the old age dependency ratio. In regression 3, we included the variable DEM (dependence rate 0-15). This was not significant for public pension purposes although the sign is negative. In regression 4, we have inserted the variable DEM (Population 65+), which like the other variables relating to the elderly is significant at the 1% threshold. In regression 5, we included the variable DEM (population 80+) which, not surprisingly, is positively and significantly correlated at the 1% level. In regression 6, we included the variable activity rate 65-69 (ACTIVERATE 65-69), which reflects the number of people aged between 65 and 69 who are still in employment. The result reveals a positive significance at the 1% level, which would mean that people in activity increasing their income through work contributes to the improvement of fiscal policy via the taxes paid and this contributes to increase public pension expenditures. This is the reason why many countries would like to reform their pension systems by keeping people in the labor market as long as possible, at least those who still have the capacity to stay there. In regression 7, we included the variable labor productivity (PROD). The result gives us a weak significance at the 10% threshold, which reveals that labor productivity is positively associated with pension expenditures. In fact, in a situation of population ageing, the government's option remains to encourage productivity improvement. This is what Hartwig (2008), said about the costs of health care that we can also include in pensions. He showed that productivity improvements lead to wage improvements, which in turn, allow for sufficient tax revenues and a proportional increase in public expenditures, including pensions.

In addition to pensions, we conducted a study on health care in relation to interest charges, the results of which are shown in Table 4.

Table 4: Regression on health spending

Dependent Health	Var:	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5	Reg. 6	Reg. 7
LagHealth		0.768*** (0.039)	0.792*** (0.039)	0.838*** (0.033)	0.766*** (0.039)	0.801*** (0.035)	0.772*** (0.038)	0.752*** (0.049)
LagDebtCharge		-0.116*** (0.033)	-0.120*** (0.036)	-0.118*** (0.034)	-0.116*** (0.033)	-0.100*** (0.034)	-0.115*** (0.032)	-0.093*** (0.040)
DEM(Rate 65+/15-64)		0.026** (0.010)					0.028** (0.011)	0.022 (0.013)
LagGdpGrowth		0.017*** (0.005)	0.016*** (0.005)	0.018*** (0.005)	0.018*** (0.005)	0.017*** (0.005)	0.017*** (0.005)	0.013** (0.005)
GenExp		0.076*** (0.008)	0.076*** (0.008)	0.074*** (0.008)	0.076*** (0.008)	0.075*** (0.008)	0.075*** (0.008)	0.065*** (0.006)
LagSaving		0.027** (0.010)	0.031*** (0.010)	0.033*** (0.011)	0.026** (0.010)	0.030*** (0.010)	0.028** (0.010)	0.028** (0.012)
DEM (Rate0-15and 65+/15-64)			0.021** (0.009)					
DEM (Rate 0-15 / 15-64)				0.001 (0.015)				
DEM (POP 65 +)					0.049*** (0.018)			
ActivityRate 65-69						0.012 (0.007)		
LaborProd							-0.111 (0.256)	
TechMed								-0.000 (0.006)
Costante		-1.939*** (0.466)	-2.592*** (0.608)	-1.877** (0.804)	-2.059*** (0.478)	-1.779*** (0.482)	-1.876*** (0.506)	-1.238*** (0.494)
Observations		601	601	601	601	597	601	448
R-sq		0.888	0.887	0.883	0.888	0.884	0.888	0.869
NumberCountry		32	32	32	32	32	32	30

Robust Standard Errors in brackets; *** p<0.01, ** p<0.05, * p<0.1

The regressions conducted in the analysis of health care yielded equally important results as the regressions conducted on pensions. The results of our basic model are listed in regression 1. Not surprisingly, the lagged dependent variable LagHealth (HEALTH) is positively and significantly correlated with the dependent variable. In fact, last year's expenditures on health care determines the current year's spending. Our key variable LagDebtCharge (DEBTCHARGE) in relation to health expenditures is significant at the 1% level of confidentiality. The signs are identical to the analysis on pensions. This reflects that health care costs would be crowded out if debt service costs were increasing. This is a strong signal in a context where OECD countries are experiencing an increase in the number of elderly people, which will be further accentuated in the coming decades. It is therefore important, as we recommend to the authorities, to take advantage of low market interest rates to work on accumulating primary surpluses that can reduce the public debt to GDP ratio and ipso facto the debt burden. Our second variable DEM (Rate 65+/15-64) is also significantly positive at the 5% level as is the regression on pension costs. It is true that the work of Dormont (Dormont (2010 and 2012), Dormont and Huber (2009), Dormont et al. (2006)), has long shown that health care costs are weakly explained by the ageing of the population, but we find in our sample a significance of 5%. This shows that the increase in the number of elderly people will certainly lead to additional costs, especially in long-term care. The variable LagGdpGrowth (GDPGROWTH) is significantly positive, which is quite the opposite of the result found for pensions. We retain this result because we believe that health care is primarily a question of government spending. Moreover, in this context, an increase in GDP leads to an improvement in salaries and, as a result, fiscal policy improves, giving the authorities the possibility for manoeuvre to increase spending on health care. In any case, improving living conditions through better health care is a plus for productivity. This is the conclusion reached by Jochen Hartwig (2008). The GenExp variable (GENEXP) is significant and positive at the 1% level. This result is consistent with our analysis. In fact, an increase in the public budget devoted to general expenditures leads to an increase in health care expenditures. The variable LagSaving (SAVING), is significant at the 5% level and positively correlated, which is contrary to the result obtained in the pension regression. We maintain this result because we believe that savings, even if counted at the aggregate level, are primarily individual savings. And, once the individual increases his savings, he is more inclined to invest in his personal health and also willing to increase the

taxes that will ultimately be used for public spending on health care. In regression 2, we included the variable DEM (Rate 0-15 and 65+/15-64) to capture the effects of the young and the elderly population on health expenditures. We find that this is significant at the 5% level, showing that the accumulation of these two variables is a heavy burden on the health system and if there is not enough labor available or good productivity, the budget deficit could increase further. Taken separately, regression 3 where we highlight the youth dependency ratio DEM (DepRatio 0-15), did not show any significance. Subsequently, regression 4 allowed us to single out the population aged 65+ (DEM (Pop 65+)). Unsurprisingly, as studies on health care demands show (Bogaert and Bains (2003)), the older ages are the ones where care is needed to cope with age-related diseases (diabetes, Alzheimer's, etc.), so the care burden is likely to increase if this category of people is growing. Hence better policies to encourage people who are still able to work to remain in the labor market, which would increase workers taxation and allow better financing of such cares. This is why in regression 5 we wanted to capture the effect of people in employment between 65 and 69 years (ACTIVERATE 65-69) on health care. This did not show any significant effects. In regressions 6 and 7, we have highlighted labor productivity (PROD) and medical technology (TechMed). The results of these regressions are not significant. The case of medical technology is particularly interesting. For Westerhout (2006), we know that technological development implies an increase in health-related expenditures, while Dormont (2009), mitigates this by insisting on the type of technological innovation (diffusion effect and substitution effect). The former increases costs and the latter reduces them. This is to show that the impact of technology is quite ambiguous. Nevertheless, in our analysis it has a negative sign. If it were significant, this would imply that CT scanners contribute to lowering health care costs, especially in medical scanners.

4.4 A cross sectional analysis

Given the fact that our regressions showed some ambiguity in the above regression results. The regression of pensions on debt charges showed insignificance, while the regression of health care proved to be significant. In this section, we have run cross-sectional regressions to interpret and analyze the results. This was done in order to also take into account the time factor, which is represented here by years. Our models here are quite simple. We regress our main variables (PENSION and HEALTH) on the public debt charges (DEBTCHARGE). Our equations have the following forms:

$$PENSION(i,t) = \beta_1 DebtCharge(i,t) + \mu(i,t) \quad (5)$$

$$HEALTH(i,t) = \beta_1 DebtCharge(i,t) + \mu(i,t) \quad (6)$$

The results are presented in Tables 5 and 6. These results are fairly standard in both regressions. In table 5, the observed signs are opposite to the results of the panel regressions. This would imply that the more the debt burden increases automatically the more the pension expenditures increase. This could also be an avenue for further analysis. Indeed, one could imagine that when the state gets into debt and therefore increases the debt charges, it is to face the pension expenses due to the implicit debt or because the national income could not cover it. It is then possible to have such a result, which is verified over the 20 years of our analysis. The same is observed in the regressions in Table 6 relating to health care expenditures to debt charges. The positive significances are indicative of an ambiguity. For in the panel analysis, the regression of these variables was found to be negatively significant. This is also an issue to be taken into account in subsequent analyses. Keynesian theories supporting an expansive fiscal policy can explain such a result. Indeed, in order to revive the economy, investment made for the purposes of growth and productivity generating positive externalities, allow states to go into debt to finance health care. For an improvement in collective well-being, through better health care for the elderly, for example, would allow them to remain active longer. In this context, the tax base would be broadened, thus allowing for a benefit in society thanks to multiplier effects. Our regression is therefore still ambiguous and needs to be analyzed further.

Table 5: Pension and debt charges regression

Years	Dependent VAR.	DebtCharge		Obs.	R-squared
2000	PENSION	2.495***	(0.331)	31	0.654
2001	PENSION	2.581***	(0.363)	31	0.627
2002	PENSION	2.476***	(0.401)	31	0.560
2003	PENSION	3.090***	(0.448)	31	0.614
2004	PENSION	3.119***	(0.465)	31	0.600
2005	PENSION	3.207***	(0.493)	31	0.585
2006	PENSION	3.323***	(0.525)	31	0.572
2007	PENSION	3.099***	(0.547)	31	0.517
2008	PENSION	3.011***	(0.577)	32	0.468
2009	PENSION	3.342***	(0.593)	32	0.506
2010	PENSION	3.544***	(0.537)	32	0.584
2011	PENSION	3.289***	(0.474)	32	0.608

2012	PENSION	3.506***	(0.488)	32	0.625
2013	PENSION	3.652***	(0.536)	32	0.599
2014	PENSION	3.676***	(0.562)	32	0.580
2015	PENSION	3.831***	(0.621)	32	0.551
2016	PENSION	4.061***	(0.683)	31	0.541
2017	PENSION	3.995***	(0.709)	32	0.506
2018	PENSION	5.010***	(0.743)	28	0.627
2019	PENSION	5.900***	(0.980)	23	0.623
2020	PENSION	6.798***	-1.549	12	0.637

Robust Standard Errors in brackets; *** p<0.01, ** p<0.05, * p<0.1

Table 6: Health care spending and debt charges regression

Years	Dependent VAR	DebtCharges		Obs.	R-squared
2000	Health	2.037***	(0.296)	32	0.605
2001	Health	2.139***	(0.335)	32	0.568
2002	Health	2.134***	(0.367)	32	0.521
2003	Health	2.708***	(0.423)	32	0.569
2004	Health	2.744***	(0.449)	32	0.546
2005	Health	2.799***	(0.481)	32	0.522
2006	Health	2.809***	(0.532)	32	0.473
2007	Health	2.549***	(0.558)	32	0.403
2008	Health	2.456***	(0.578)	32	0.368
2009	Health	2.934***	(0.559)	32	0.471
2010	Health	3.061***	(0.517)	32	0.530
2011	Health	2.716***	(0.476)	32	0.512
2012	Health	2.943***	(0.471)	32	0.557
2013	Health	2.981***	(0.512)	32	0.522
2014	Health	2.987***	(0.534)	32	0.502
2015	Health	3.129***	(0.592)	32	0.474
2016	Health	3.354***	(0.660)	31	0.463
2017	Health	3.318***	(0.684)	32	0.431
2018	Health	3.632***	(0.749)	32	0.431
2019	Health	3.895***	(0.840)	32	0.409
2020	Health	3.663**	(-1281)	20	0.301
Robust Standard Errors in brackets; *** p<0.01, ** p<0.05, * p<0.1					

5. Conclusion and recommendations.

Spending on pensions and health care are undoubtedly very important pockets of public expenditures for the social well-being of OECD countries. Studies in this area show a considerable increase in the coming decades if nothing is done, due to the ageing of the population. The objective of this paper was to evaluate whether the interest charges could be a threat for the "said" ageing expenditures. In our study, we have highlighted debt burden in an attempt to answer the question of whether these can be crowding out pension and health care spendings. The empirical results showed that debt charges are ambiguous in the current context. Indeed, the ordinary least squares analysis did not show a certain homogeneity in the tests performed. The tests on pension expenditures were not significant with the panel data. The negative sign nevertheless showed that if they were to be significant, these charges would crowd out pension expenditures. Tests on health care costs have shown some significance in panel data. And logically, this would mean that in the event of an increase in public debt charges, health care spending would be threatened. However, as the two regressions did not give a certain homogeneity in the results, we proceeded to cross-sectional regressions to take into account the time factor and analyse the results. We have noted that the results are homogeneous in both settings. But the signs obtained were contrary to the regressions carried out in the panel. Hence the ambiguity in our results. we can say, however, that given the current low interest rates on the market reflecting in the debt charges, we can say that these would not currently threaten the "said" ageing expenditures. On the other hand, demographic variables such as the ratio of elderly people, the population over 65 and 80 years old showed a strong correlation in both pension expenditures and health care. This shows that the latter are likely to put additional pressure on pensions and health care. Hence the call for reforms to encourage people to stay in work for a long time in sectors where this is possible. Also, the integration into the labor market of immigrants for whom this is often difficult to access could also be a solution. Also, reforms, especially in pension systems, should be made in the sense of rationing expenditures, taking into consideration future generations; this is a country by country analysis. In health care, improved medical technology should go hand in hand with improved care and cost cutting. This is mainly a question of reviewing patent policies. In the end, we believe that the burden on the public debt as said above, should not be a threat to the sustainability of ageing expenditures. At most, the increasing old-age and senior dependency ratio should be the main concern of policy makers. In the context of ageing, policies would take advantage of low

debt burdens to direct spending in policies that would improve the sustainability of ageing expenditures. Policies to improve human capital could be an example. Spendings on education and training are strongly recommended, especially in sectors with high capital returns, such as technology and innovation sectors, research and development and so on. A well-trained human capital could improve overall productivity, which in turn would generate economic growth, thus making it possible to meet the costs of pensions and health care. Shouldn't we think about investing in human capital through education and training to boost wealth-generating productivity?

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Modifications on Book-Valued Ratios

Catherine Georgiou^{1, †}

¹ Department of Economics, Business Division, Aristotle University of Thessaloniki, 54124, Thessaloniki, Greece

ARTICLE INFO	ABSTRACT
Article History	Purpose:
Received 20 December 2022; Accepted 7 February 2023	In this paper we try to explain US stock market variations and cash flow fundamentals by employing three different book-valued based ratios. First, we explore the explanatory capacity of the simple book-market ratio on time-varying expected returns, and proceed on altering its construction so as to enhance its performance. We then run the extra mile by constructing two new ratios, the book-dividends and book-earnings ratios based on the long-run equilibrium relationships between book, dividends and earnings. Our analysis includes evidence of predictability on dividend and earnings growth rates on the S&P 500 for the most recent sample period 1926-2018. We also investigate the ratios' forecastability by sub-sampling.
JEL Classifications G11, G12, G13, G14, G17	Design/methodology/approach:
	We commence our analysis with the conventional book-market (bm) ratio and by failing to reject the hypothesis of a unit root, we propose the modified book-market (mbm) ratio, whose construction is based on the long-run equilibrium relationship between book (b) and market (m) values. We proceed on associating book values to dividends and earnings series and fix the book-earnings (be) and the dividend-book (db) ratios. We similarly modify be and db, and examine their forecasting performance on returns, dividend and earnings growth.
	Findings:
	In-sample evidence suggests that an investor who employs mbm can improve its forecasts by 37% and 41% in the 7- and 10-year return horizon, while the modified dividend-book (mdb) proves even more beneficial by explaining 53% and 59% in similar return horizons. Our modified book-earnings (mbe) has a very good in-sample fit to the earnings growth data unlike the rest of the predictors. With respect to the out-of-sample performance, mbm manages to surpass the simplistic forecast benchmark only at the 10-year horizon by 15% while mdb attains an impressive R^2_{oos} of 47% and 71% at the 7- and 10-year return horizon.
	Research limitations/implications:
	Further research is required so as to solve the earnings puzzle in terms of forecasting along with the necessity to understand the economical sources behind non-stationarity in valuation ratios.
Keywords: book-market ratio, modified book-market ratio, book-valued ratios, non-stationary ratios, modified ratios, return predictability	Originality/value:
	We believe that our paper may prove enlightening to investors focused on portfolio allocation and asset pricing and scholars interested in return forecasting, capital budgeting and risk identification.

1. Introduction

The empirical literature on stock return predictability includes a large number of financial variables with the capacity to predict future stock returns. To name but a few, the dividend-price ratio (Fama and French, 2002; Cochrane, 2008), the price-earnings ratio (Lamont, 1998; Campbell and Shiller, 2001), the book-market ratio (Kothari and Shanken, 1997; Pontiff and Schall, 1998), the dividend-earnings ratio (Lamont, 1998), the term and default spreads on bonds (Welch and Goyal, 2008), as well as the consumption-wealth ratio (Lettau and Ludvigson, 2001) are only some

[†]Corresponding Author: Catherine Georgiou
Email: ageorgiou@econ.auth.gr

indicative examples of the most renowned variables with evident forecastability. Despite the econometric limitations related to either overlapping observations or the lack of exogenous regressors in the predictive regression models (Nelson and Kim, 1993; Stambaugh, 1999), Campbell (2000, pg. 1523) comments that *“the evidence of predictability survives at reasonable if not overwhelming levels of statistical significance. Most financial economists appear to have accepted that aggregate returns do contain an important predictable component”*.

Since the focus of the present study is the book-market (bm) ratio, evidence that relate return predictability with bm is originated in the studies of Fama and French (1992, 1993) who find that bm can explain variations in cross-sectional data. Later studies of Davis (1994) and Chan et al. (1995) follow, reconfirming the forecastability of bm, while Kothari et al. (1995) and Loughran (1996) argue that both the significance and the magnitude of the ratio's predictive capacity may depend on data mining and various biases in the database employed. Cochrane (1999) supports that it is the prices rather than the book values that determine any forecastability of the ratio; low price-book values today are a signal of high average returns tomorrow, thus regardless whether we study individual stocks or sort them into portfolios, book values alone express minor predictive ability. On the other hand, strong evidence of the ability of bm to forecast returns on time-series data is primarily reported by Kothari and Shanken (1997) and Pontiff and Schall (1998). Kothari and Shanken (1997) compare the predictive capacity of the dividend yield to bm and conclude that the latter is a stronger predictor in their full sample. Also, Pontiff and Schall (1998) support that bm predicts market returns and attribute this capacity to the fact that book values proxy for expected cash flows. Their main rationale is that provided that cash flow is constant, then an increase in the discount rate leads to a decrease in the market value and consequently, an increase in bm ratio. This may explain the positive relation between future returns and present bm ratios. In their study they construct two bm ratios, based on either S&P or DJIA book values, and find that the S&P bm ratio is by far a better forecaster of market returns, even when they sub-sample. Finally, they relate this forecastability to the capacity of book value to forecast cash flows by retrieving cointegration relations between earnings and book values of the two indexes.

There are two critical issues that characterize forecasting superiority of the indicated valuation ratios; persistency and stationarity. With regards to the first issue, the more persistent the variables, the stronger the forecasts we receive particularly when we extend the forecasting horizon (see the discussion in Campbell and Viceira, 2002; Campbell and Yogo, 2006; Chen, 2009). The stationarity concern on the other hand, is a more complex issue. Traditionally, valuation ratios with predictive capacity have been treated as stationary processes in adherence to standard economic principles. The basis of this assumption though is rather fragile since it lies on the argument that both the data and the horizons, we examine are infinite. However, in practice this argument does not hold since data and horizons are well specified in all studies. Consequently, in an attempt to be more “pragmatic”, we can sideline the stationarity issue and examine the presence of long-run equilibrium relations among the pairs of series on which valuation ratios such as the dividend and earnings yields, and book-market are constructed. In fact, the existence of cointegration vectors in the aforementioned ratios is no news to empirical finance. Froot and Obstfeld (1991) fail to reject the null hypothesis of a unit root between dividends-prices, dividends-earnings and price-earnings. Also, Pontiff and Schall (1998) retrieve evidence of a cointegration vector between earnings and book values and try to find the source of predictability in S&P and DJIA data. Research efforts expand on tri-variate vectors as well, with the most indicative examples the cay and cdy variables by Lettau and Ludvigson (2001, 2005) and the dpe by Garrett and Priestley (2012). More specifically, evidence is presented that the long-run relations between consumption (c), asset wealth (a) and income (y) on the one hand, and instead of asset wealth, dividends (d) on the other, can provide substantial forecasts on US returns and dividend growth. Similar findings are reported by Garrett and Priestley (2012) who construct a strong predictor based on the cointegration among dividends (d), prices (p) and earnings (e). More recently, Polimenis and Neokosmidis (2016, 2019) focus on the forecasting behavior of the dividend yield and conclude that by fixing its modified version, (which essentially stands as the stationary trend deviation between dividends and prices) they can provide significant improvements in return forecasting patterns.

Motivated by these findings, the present study attempts to (a) construct a stationary modified book-market (mbm) ratio, (b) explore the cointegration relation of book values to dividends and earnings and (c) examine the predictive ability of these book-valued ratios compared to their modified counterparts. More specifically, we report that we cannot statistically reject the hypothesis of a unit root in bm and proceed on forming its modified counterpart based on the long-run equilibrium relationship between logged book and market values. Our efforts focus on de-noising the simple bm ratio with the hope of tackling with some of its forecasting inabilities. We also isolate dividend, earnings and book values and test for cointegration relations. We find that similarly to bm, there are two cointegration vectors in book values and earnings $[b \ e]$ and in dividends and book values $[d \ b]$ and form their modified versions as well. Our simple book-valued ratios, namely bm, be and db are all tested for their forecastability alongside with their modified versions on high quality S&P 500 annual return, dividends and earnings growth data.

The main findings are that (a) the modified bm (mbm) has a better return in-sample fit over the traditional bm, (b) the modified db (mdb) provides substantial forecasting improvements compared to the rest book-valued ratios, explaining 59% of total return variations in-sample, and (c) our book-earnings (be) ratio is able to reveal better the forecasting patterns in earnings growth. Regarding the out-of-sample (oos) outcomes (a) our mbm is able to surpass the simplistic forecast benchmark at the longest horizon in contrast to bm, (b) both be and mbe do not generalize well thus further research is needed to comprehend this extra complexity in earnings-ratios composition and (c) an investor who employs our mdb is able to enhance his forecasts by 47% and 71% at the 7- and 10-year return horizon.

The main contribution of the present study is to re-evaluate the predictive capacity of the simple book-market ratio on S&P 500 data and extend the analysis by revealing its forecastability, if any, in dividend and earnings growth. By slightly altering the conventional composition of bm through employing a stationary trend deviation between logged

book and market values, we manage to improve the ratio's forecasting benefits. Additionally, by fixing ratios (both simple and modified) based on the long-run equilibrium relations between logged dividend and earnings to book values, we present new evidence of enhanced predictability in the empirical literature. We believe that our paper may prove enlightening to investors focused on portfolio allocation and asset pricing and scholars interested in return forecasting, capital budgeting and risk identification.

The rest of the paper is organized as follows. In the next subsection, we discuss the necessity to study the non-stationarity of the simple ratios and form their modified versions. Section 2 presents the data and stresses on the methodology followed to estimate the stationary trend deviation between book and market (or dividends, or earnings) values. In section 3 in-sample and out-of-sample predictability findings are discussed. Section 4 includes the concluding remarks.

1.1 Non-stationary book-valued ratios

The vast majority of the studies in the field consider valuation ratios similar to this paper stationary and base this assumption on the infinity of the samples and the forecasting horizons. In reality though, both the size of the samples and the horizons we examine (either in the short or the long run) are well-specified, let alone when statistical tools are used, they cannot reject the hypothesis of the existence of a unit root (see the discussion in Lamont, 1998; Goyal and Welch, 2003; Lettau and Ludvigson, 2001, 2005 among others). Consequently, non-stationarity and persistency are strong traits of the series that comprise dividend and earnings yields, and as we will show later book-market ratio as well.

Table 1a: Correlation matrix and descriptive statistics.

	r_t	re_t	rr_t	rf_t	bm_t	mbm_t	Mean	Std	AR (1)
r_t	1						0.09	0.19	0.06
re_t	0.99	1					0.06	0.19	0.06
rr_t	0.98	0.99	1				0.06	0.19	0.02
rf_t	0.07	-0.09	-0.03	1			0.03	0.03	0.90
bm_t	-0.16	-0.17	-0.20	0.15	1		-0.71	0.52	0.91
mbm_t	-0.16	-0.22	-0.23	0.42	0.66	1	0.33	0.35	0.83

Note: We present the descriptive statistics for annual nominal (r_t), excess (re_t) and real returns (rr_t), risk-free rates (rf_t), book-market (bm_t) and the modified book-market (mbm_t). The table shows the correlation matrix among the series, as well as the mean, standard deviation and the autocorrelation coefficient based on AR (1) fitted model. Data is annual from 1926-2018.

Table 1b: Correlation matrix and descriptive statistics.

	r_t	re_t	rr_t	rf_t	be_t	mbe_t	db_t	mdb_t	Mean	Std	AR (1)
r_t	1								0.09	0.19	0.06
re_t	0.99	1							0.06	0.19	0.06
rr_t	0.98	0.99	1						0.06	0.19	0.02
rf_t	0.07	-0.09	-0.03	1					0.03	0.03	0.90
be_t	-0.20	-0.19	-0.17	-0.08	1				2.03	0.36	0.62
mbe_t	-0.20	-0.20	-0.18	0.04	0.90	1			2.22	0.31	0.55
db_t	-0.03	0.01	0.00	-0.21	0.35	-0.08	1		-7.37	1.76	1.00
mdb_t	0.05	-0.01	-0.01	0.37	0.15	0.18	0.13	1	-2.47	0.29	0.87

Note: We present the descriptive statistics for annual nominal (r_t), excess (re_t) and real returns (rr_t), risk-free rates (rf_t), book-earnings (be_t), the modified book-earnings (mbe_t), dividend-book (db_t) and the modified dividend-book (mdb_t) ratios. The table shows the correlation matrix among the series, as well as the mean, standard deviation and the autocorrelation coefficient based on AR (1) fitted model. Data is annual from 1926-2018.

As shown in descriptive statistics in Table 1a, the book-market ratio has an autocorrelation coefficient $\phi=0.91$ implying that unit root tests may not have enough power to tackle with such high ϕ values. Following the same line of thought, we fix two new variables by associating the logged 12-month summed up earnings series (e) with log book values (b) on one hand, and the logged 12-month summed dividends (d) with log book values (b) on the other¹. As in the case of book-market (bm), the book-earnings (be) and the dividend-book (db) are also highly persistent with autocorrelation coefficients that reach the values of 0.62 and 1.00 respectively as depicted in Table 1b. As it is commonly accepted, true persistence in finite samples tends to be highly underestimated by typical estimation methods. In this study, we mainly use two ways in rejecting stationarity; we firstly use a straightforward ADF test, and we secondly impose a restriction on the cointegration vector $[b_t \ m_t]$ (also for $[b_t \ e_t]$, and $[d_t \ b_t]$) as summarized in Panel B of Tables 2a, 2b and 2c. Moreover, in the Appendix we cite more robust econometric evidence against the stationarity issue of not only the conventional bm but also of our newly formed be and db.

We proceed our analysis by presenting evidence of long-run equilibrium relationships among three pairs of series, namely (b-m), (b-e) and (d-b) based on the Johansen technique (1991). By imposing a strict restriction on the cointegration vectors $[b \ m]$, $[b \ e]$ and $[d \ b]$, we reject the hypothesis in all three vectors that the logged values of

¹ See section 2.1 of the present study for more details on the ratios' construction.

each pair are linked with long run equilibrium relations of the form b-m (b-e and d-b respectively). Tables 2a, 2b and 2c present the results.

Table 2a: Cointegration test for the $[b_t \ m_t]$ vector and the null hypothesis of $[1 \ -1]$.

Panel A	#Coint. Vector	Trace test stat.	5% critical value	
	0	29.79*	0	20.26
	≤ 1	8.38	≤ 1	9.16
Panel B	$H_0: [1 \ -1]$	χ^2 -stat. 5.14*		

Note: In Panel A we apply the Johansen testing process, assuming no deterministic trend on the cointegration relationship. The pair $[b_t \ m_t]$ tests for a cointegration relationship between the book (b) and the market values (m). Panel B presents the results for the restriction test that $[1 \ -1]$ spans the cointegration space among (b, m). (*) and (**) denote the 5% and 1% rejection level respectively. Data are overlapping annual for the period 1926-2018.

Table 2b: Cointegration test for the $[b_t \ e_t]$ vector and the null hypothesis of $[1 \ -1]$.

Panel A	#Coint. Vector	Trace test stat.	5% critical value	
	0	20.55*	0	15.49
	≤ 1	0.01	≤ 1	3.84
Panel B	$H_0: [1 \ -1]$	χ^2 -stat. 4.62*		

Note: In Panel A we apply the Johansen testing process, assuming trending series and no trend on the cointegration relationship. The pair $[b_t \ e_t]$ tests for a cointegration relationship between the book (b) and the 12-month summed-up earnings (e). Panel B presents the results for the restriction test that $[1 \ -1]$ spans the cointegration space among (b, e). (*) and (**) denote the 5% and 1% rejection level respectively. Data are overlapping annual for the period 1926-2018.

Table 2c: Cointegration test for the $[d_t \ b_t]$ vector and the null hypothesis of $[1 \ -1]$.

Panel A	#Coint. Vector	Trace test stat.	5% critical value	
	0	23.47*	0	20.26
	≤ 1	8.93*	≤ 1	9.16
Panel B	$H_0: [1 \ -1]$	χ^2 -stat. 8.89*		

Note: In Panel A we apply the Johansen testing process, assuming no deterministic trend on the cointegration relationship. The pair $[d_t \ b_t]$ tests for a cointegration relationship between the 12-month summed-up dividends (d) and the book values (b). Panel B presents the results for the restriction test that $[1 \ -1]$ spans the cointegration space among (d, b). (*) and (**) denote the 5% and 1% rejection level respectively. Data are overlapping annual for the period 1926-2018.

More recent evidence suggests that we should treat dividends and earnings-related ratios as non-stationary because they do include trends (therefore, contradicting the so far established notion that stock prices and corporate fundamentals are highly associated). The entire concept of the dependence relations between corporate dividend and earnings on the one hand, and earnings and stock prices on the other, is now overturn since the data itself is arbitrary and not as linked to asset prices as econometricians traditionally have expected. This perception is further strengthened by statistics, such as the ADF test which cannot reject the presence of a unit root, indicating that the entire finance society should re-consider the non-stationary dynamics of ratios related to these series.

Consequently, the next logical step is to sideline the stationarity concern and proceed in searching for cointegration relationships among the valuation series. We begin by assuming a deterministic long run equilibrium relationship between book (b) and market (m) values, that is a cointegration vector that follows the form:

$$b_t = \alpha + \beta m_t \quad (1)$$

We similarly treat book (b) and 12-month summed-up earnings (e), and the 12-month summed-up dividends (d) and book (b) values. By then allowing the data to unfold the true cointegration vector of the form $[1 \ -\beta]$ in eq. (1), we fix our modified book-market ratio (mbm) as the stationary cointegration errors of this long-run equilibrium which is expressed as:

$$mbm_t = b_t - \beta m_t \quad (2)$$

The β coefficient in eq. (2) stands as the unique population parameter that harmonizes the relation between book and market values via revealing the stationary trend deviations between them. It essentially manages to express the drift differential between the two series, and if it receives a value lower to unit then it must express the lower growth rate of book values against market values. We believe that the modified book-market (mbm) is a more reliable forecaster compared to the non-stationary bm which includes a small noise trend. Since mbm is highly persistent with an AR (1) coefficient at $\phi=0.83$ (as reported in Table 1a) then its predictive capacity must surpass the short-term horizons and extend in the long-run.

Similarly, the long run relation between b and e, and d and b are studied and we define the modified book-earnings and dividend-book ratios as the stationary cointegration errors of the following long-run equilibriums:

$$mbe_t = b_t - \gamma e_t \quad (3)$$

$$(4) \quad mdb_t = d_t - \delta b_t$$

The rationale remains unaltered since if mbe and mdb show short-horizon forecastability then with autocorrelation coefficients at 0.62 and 1.00 (as reported in Table 1b), they retain the persistency trait strong enough to predict returns both in the short and (perhaps the most interesting) long-run.

Based on annual data, we present evidence that an investor who uses the modified book-market ratio (mbm) can enhance his forecasting in-sample by 32%, 37% and 41% at 5-, 7- and 10-year horizons (medium, medium-to-long, long horizons) against the equivalent values of 16%, 23% and 31% of the traditional book-market (bm) ratio. Furthermore, our modified dividend-book ratio (mdb) is able to predict 38%, 53% and 59% of returns in-sample in similar horizons and provides an astonishing R^2 of 47% and 71% at the 7- and 10-year horizons ahead out-of-sample. The classical bm but also be and db may reveal some predictive capacity, but only in-sample and of lesser magnitude compared to their proposed modified counterparts.

2. Empirical Methodology

This study exploits high quality return data for the S&P 500 index, with and without dividends, as available by CRSP since 1926. Our full sample² spans the most recent 93-year period including values from January 1926 to December 2018. We also proceed on examining the ratios' forecastability on the pre and post-1965 sub periods. Nominal data have primarily been used³ since forecasting in long horizons is highly dependable on whether we use real or nominal returns and dividend growth equivalently (see the discussion in Engsted and Pedersen, 2010).

2.1 Construction of the conventional book-market ratio

The book-market ratio (bm) is essentially the ratio of book to market values and is given by the formula:

$$(5) \quad bm_t = b_t - m_t = \log(B_t/M_t)$$

The ratio's computation for the months January and February includes the division of book value at the end of two years ago by the price at the end of the current month, while from March to December book value is divided at the end of the previous year by the price at the end of the current month. The data set is similar to other studies that investigate the ratio's ability to forecast returns (see for instance, Goyal and Welch, 2008; Pontiff and Schall, 1998; Kothari and Shanken, 1997).

In an attempt to surpass the observed seasonality in dividend and earnings series when a monthly frequency is used (see for example Chen, 2009), we prefer the annual horizon in fixing the book-earnings and the dividend-book ratios. Firstly, the cointegration is recovered from all series on a monthly frequency and secondly, we modify them accordingly and sample them on an annual horizon so as to match the rest of our econometric analysis. While working on earnings series is straightforward⁴, extracting dividends is a more complex issue. There are two key points we need to consider; first, whether the re-investment assumption will be taken into consideration⁵ and second, if a simpler and more representative approach on forming the dividend series will be used so as to capture more accurately decision making when it comes to dividend setting schemes in enterprises. In this paper we follow the second approach and extract dividends from monthly gross returns (R_t) where $R_t = \frac{P_t + D(t)}{P_{t-1}}$, and monthly returns due to price gain alone (that is excluding dividends, X_t) where $X_t = \frac{P_t}{P_{t-1}}$. Therefore, dividends at month t follow the form of:

$$D_{(t)} = \left(\frac{R_{(t)}}{X_{(t)}} - 1 \right) * P_t \quad (6)$$

Regarding the notation, $D_{(t)}$ is the monthly dividend for month t , while D_t is the ending at month t annual dividend. So, at the annual frequency the annualized dividend computation is given by the formula $D_t = \sum_{i=0}^{11} D(t-i)$. Having calculated annual dividends, the computation of dividend-book ratio is given by:

$$db_t = d_t - b_t = \log(D_t/B_t) \quad (7)$$

Similarly, the book-earnings ratio is estimated as:

² We retrieve our data set from Goyal's database, available at <http://www.hec.unil.ch/agoyal>.

³ We have also examined excess and real return predictability with no significant differences in the outcomes.

⁴ Earnings data is retrieved by <http://www.econ.yale.edu/~shiller/data.htm>.

⁵ In fact, Chen (2009) argues that reinvested dividends absorb much of the market's volatility for the year, and they may thus tangle with true cash availability to the shareholders.

$$(8) \quad be_t = b_t - e_t = \log(B_t/E_t)$$

2.2 The modified book-valued ratios

In this sub-section we describe the econometric methodology followed so as to construct our modified book-market ratio (mbm). Similar steps are followed so as to modify book-earnings (be) and dividend-book (db) ratios.

In order to test for cointegration we base our analysis on the Johansen approach (1995a) which basically examines the number of eigenvalues that are statistically different than zero. The implementation of the approach involves several steps which we describe below.

First, we need to consider a two-dimensional vector $v_t = [b_t \ m_t]'$ and assume that a cointegrating vector c is present. Thus, $c'v_{t-1}$ represents the error in the data set and quantifies at $t-1$ the extent at which the series deviate from the stationary mean. By examining error correction, we can check the tendency of the cointegrated series to return back to a common stochastic trend. As a result, this trend deviation from equilibrium in the long-run between book and market values (book and earnings, dividends and book values equivalently) helps us derive the modified bm (mbm) which follows the form of eq. (2). The same applies for the modified be (mbe) and the modified db (mdb) of eq. (3) and (4) respectively.

The disequilibrium that mbm contains is corrected by the book and market values at a rate that a vector of their own adjustment speed α captures. Consequently, a multiplicative error-correction term $\alpha c'v_{t-1}$ is formed which we need to consider to a simple VAR model so as to jointly interpret book and market change (Δb and Δm) and generate a VEC(w) model of the form:

$$\Delta v_t = \sum_{i=1}^w B_i \Delta v_{(t-i)} + \alpha(c'v_{t-1} + d_0) + d_1 + u_{(t)} \quad (9)$$

As usual we assume at first that all tested vectors follow the form of eq. (9), in other words we examine for deterministic cointegration relationships. However, in the cases of both $[b_t \ m_t]$ and $[d_t \ b_t]$ the data leads to assuming that there is no deterministic trend and no intercept in the data. Thus, in these case eq. (9) is transformed to the following:

$$(10) \quad \Delta v_t = \sum_{i=1}^w B_i \Delta v_{(t-i)} + \alpha c'v_{t-1} + u_{(t)}$$

The next steps include the estimation of either model 9 or 10 in a VAR in levels after assuming a maximum order of 12 lags (since cointegration is tested on series with monthly frequency) and determine the most appropriate lag length for each vector. By employing the Hannan-Quinn (HQ) criterion⁶ we conclude that 1 lag should be used for VAR and thus, zero lag for VECM in the case of $[b_t \ m_t]$. As indicated by trace statistics included in Table 2a log book and market values are cointegrated following the form of:

$$mbm_t = b_t - 0.781672m_t \quad (11)$$

Additionally, we pose extra restrictions to show that the vector $[1 \ -1]$ in each pair does not span the cointegration space. Panel B of Table 2a clearly shows through χ^2 that bm strongly behaves in a non-stationary manner and perhaps unit root tests cannot capture this behavior effectively since we are dealing with highly persistent variables.

The approach followed for the modified book-earnings (mbe) and modified dividend-book (mdb) is similar; consequently, for the vectors $[b \ e]$ and $[d \ b]$ we find that there is a long-run equilibrium relationship⁷ in each pair that follow the form equivalently:

$$(12) \quad mbe_t = b_t - 0.904375e_t$$

$$(13) \quad mdb_t = d_t + 0.210458b_t$$

We primarily focus on the Johansen test since it deals with some of the weaknesses in the Engle-Granger approach. More specifically, there are two main benefits the first of which is that we avoid the two-step procedure that the Engle-Granger technique entails (see more details in the Appendix) and second, we can pose restrictions (like the ones we report in Panels B of Tables 2a, 2b and 2c) to eliminate all doubts on cointegrated series.

⁶ There is valid reason to base the lag length selection on the HQ criterion (see the discussion in Harris and Sollis, 2003).

⁷ Findings on each vector can be found in Tables 2b and 2c.

3. Results

3.1 In-sample predictability

This section includes the primary univariate forecasting regressions based on the conventional book-market ratio (bm) and its modified counterpart (mbm). We have also enriched our analysis with book-earnings (be) and dividend-book (db) ratios and their respective modified counterparts (mbe and mdb). We use annual S&P 500 data so as to form continuously compounded returns for 3, 5, 7 and 10-year horizons ($h = 3, 5, 7, 10$) for the period 1926-2018. Our forecasting regressions follow the classical form:

$$(14) \quad r_t(h) = \alpha + c x_t + u_t(h)$$

where r_t stands as the log nominal returns at time t and horizon h each time, and x_t is one of the studied predictors each time. We form similar regressions when the left-hand variable is either dividend or earnings growth⁸. Standard errors are GMM corrected based on the Hansen-Hodrick formula.

Table 3a: In-sample predictability of nominal returns.

		b	t(b)	R^2			b	t(b)	R^2
$r_t(3)$	bm_t	0.22	2.36	0.11	$r_t(5)$	bm_t	0.32	3.03	0.16
	mbm_t	0.43	2.60	0.22		mbm_t	0.64	3.05	0.32
	be_t	0.11	0.63	0.01		be_t	0.21	1.02	0.03
	mbe_t	0.18	0.76	0.03		mbe_t	0.32	1.08	0.06
	db_t	-0.00	-0.07	0.00		db_t	-0.01	-0.12	0.00
	mdb_t	0.60	4.06	0.29		mdb_t	0.84	5.31	0.38
$r_t(7)$	bm_t	0.39	4.05	0.23	$r_t(10)$	bm_t	0.57	6.59	0.31
	mbm_t	0.71	4.60	0.37		mbm_t	0.90	7.26	0.41
	be_t	0.14	0.86	0.02		be_t	0.17	0.82	0.01
	mbe_t	0.23	0.89	0.03		mbe_t	0.22	0.71	0.02
	db_t	0.01	0.11	0.00		db_t	0.04	0.36	0.01
	mdb_t	1.02	7.32	0.53		mdb_t	1.29	13.52	0.59

Note: Standard errors are GMM corrected. Data is annual spanning the period 1926-2018.

Table 3a presents evidence on the full sample univariate outcomes for all ratios. As it is well understood in empirical literature, forecasting in longer horizons is the mechanical effect of short-horizon same direction forecastability in combination with a highly persistent forecaster (see the discussion in Campbell and Viceira, 2002; Campbell and Yogo, 2006; Cochrane, 2008). Consequently, a highly persistent predictor leads to increased slope coefficients in longer horizons. Our findings confirm these mechanics since both our slope coefficients and R^2 s increase impressively as we extend the forecasting horizon. All ratios are able to predict returns in all horizons except book-earnings (be), while the modified ratios perform even better in all the three criteria set, namely slope, t-statistics and R^2 .

More specifically, bm can predict returns in all horizons but the modified bm is able to produce better results reaching an R^2 of 37% at $h=7$ while bm can explain only 23% at the same horizon. Apart from a more enhanced performance over the classical bm, mbm manages to even surpass itself as we increase the horizon. The case is similar for both be and db, with the observed superiority of the modified ratios over return forecasting. For instance, at the 10-year horizon ahead, db can explain 1% of total return variations while mdb reaches the value of 59%. Results for b are of extremely low magnitude, contradicting Pontiff and Schall's findings (1998) who employ a similar ratio; even mbe seems unable to capture any predictive component in returns.

By directly comparing mbm with mdb, we observe that both the slope and the explanatory power (R^2) of mbm is of lesser magnitude since mdb has already reached Cochrane's theoretical limit of 1 in the medium-to-long and the longest horizons while mbm even at $h=10$ reaches the value of 0.90 (see the discussion in Cochrane, 2011). Moreover, while mbm produces an R^2 of 37% and 41% the 7- and 10-year horizons, mdb is already at 53% and 59%. In other words, the performance of mbm in the longest h is already surpassed by mdb in the medium h . In an attempt to help understand the evidence, we have isolated these findings in Table 3b.

Table 3b: In-sample predictability of nominal returns for bm and mbm vs. db and mdb.

		b	t(b)	R^2			b	t(b)	R^2
$r_t(3)$	bm_t	0.22	2.36	0.11	$r_t(5)$	bm_t	0.32	3.03	0.16
	mbm_t	0.43	2.60	0.22		mbm_t	0.64	3.05	0.32
	db_t	-0.00	-0.07	0.00		db_t	-0.01	-0.12	0.00
	mdb_t	0.60	4.06	0.29		mdb_t	0.84	5.31	0.38

⁸ The main results presented in the paper are on nominal values, even though we have examined the performance of our forecasters in both real and excess values.

$r_t(7)$	bm_t	0.39	4.05	0.23	$r_t(10)$	bm_t	0.57	6.59	0.31
	mbm_t	0.71	4.60	0.37		mbm_t	0.90	7.26	0.41
	db_t	0.01	0.11	0.00		db_t	0.04	0.36	0.01
	mdb_t	1.02	7.32	0.53		mdb_t	1.29	13.52	0.59

Note: Standard errors are GMM corrected. Data is annual spanning the period 1926-2018.

Therefore, an investor who employs mbm can interpret from 22% to 41% of future return variation for a 3-year to a 10-year horizon. However, the classical bm can explain from 11% to 31% in similar horizons. An even more powerful finding is that by employing mdb, an investor can achieve even better forecasting benefits from 29% to 59% in similar horizons, while the simple db ratio seems incapable to predict returns. Figure 1 illustrates the direct comparison of mbm to mdb in all horizons by plotting R^2 values for all horizons. The clear dominance of mdb is clearly evident regardless the horizon.

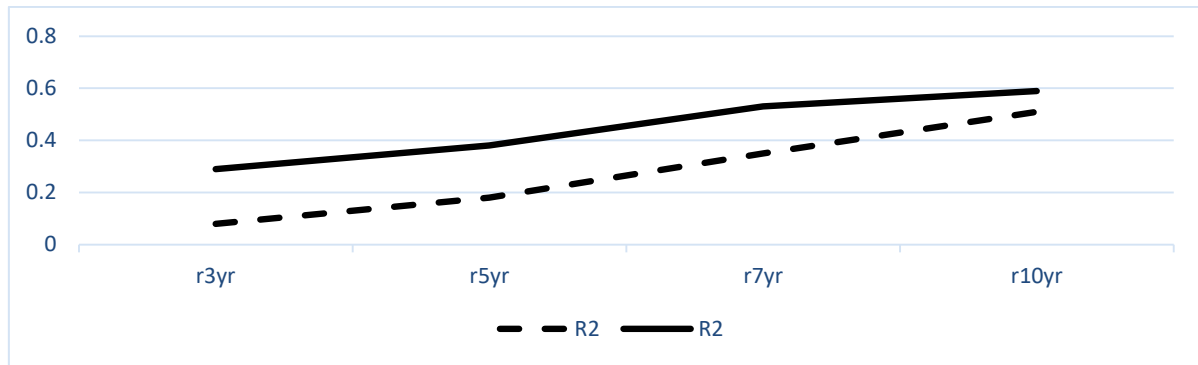


Figure 1: In-sample forecasting in horizons for annual nominal returns.

Note: The figure shows the evolution of R^2 as we extend the horizon of our forecasts for the conventional bm and its modified counterpart versus dividend-book and the modified dividend-book ratios. Data is annual spanning the period 1926-2018.

In an attempt to interpret the forecasting benefits derived by high persistency in the predictors, we observe that the high AR (1) coefficient of the modified bm at $\phi=0.83$ must be because bm includes a small, impeded unit root. Yet, the simple bm is even more persistent (at $\phi=0.91$) but this higher persistency does not relate to predictive capacity⁹. We believe that the extra forecastability of the modified over the simple ratios is attributed to this lower persistency which determines the true forecasting horizon.

Additionally, we have examined the ratios' ability to predict excess and real returns and concluded that the findings are not precisely equivalent to the nominal ones, even though the modified ratios retain their upgraded performance. We attribute this to the forecasting ability of the ratios (both conventional and modified) over the risk-free component included in total equity return on the one hand, and inflation growth included in real returns on the other¹⁰.

Table 4: In-sample predictability of dividend and earnings growth.

		b	t(b)	R^2			b	t(b)	R^2
$\Delta d_t(3)$	bm_t	0.08	1.45	0.03	$\Delta d_t(5)$	bm_t	0.13	1.50	0.05
	mbm_t	0.25	2.17	0.14		mbm_t	0.40	1.96	0.24
	be_t	0.10	3.24	0.02		be_t	0.26	2.63	0.10
	mbe_t	0.19	2.06	0.07		mbe_t	0.42	2.73	0.20
	db_t	-0.03	-1.04	0.04		db_t	-0.04	-1.17	0.06
$\Delta d_t(7)$	mdb_t	0.11	1.00	0.02	$\Delta d_t(10)$	mdb_t	0.17	1.05	0.03
	bm_t	0.14	1.32	0.05		bm_t	0.16	1.25	0.07
	mbm_t	0.42	2.38	0.24		mbm_t	0.42	2.02	0.23
	be_t	0.23	3.11	0.07		be_t	0.19	1.83	0.05
	mbe_t	0.40	3.62	0.16		mbe_t	0.32	1.75	0.10
$\Delta e_t(3)$	db_t	-0.05	-1.07	0.07	$\Delta e_t(5)$	db_t	-0.04	-0.76	0.05
	mdb_t	0.15	1.07	0.02		mdb_t	0.18	1.04	0.03
	bm_t	0.09	1.42	0.01		bm_t	0.16	1.73	0.03
	mbm_t	0.24	1.21	0.04		mbm_t	0.43	1.28	0.10
	be_t	0.64	5.29	0.25		be_t	0.78	5.44	0.35
$\Delta e_t(7)$	mbe_t	0.81	7.76	0.31	$\Delta e_t(10)$	mbe_t	1.03	5.87	0.46
	db_t	-0.02	-0.65	0.01		db_t	-0.04	-1.13	0.02
	mdb_t	0.05	0.27	0.00		mdb_t	0.11	0.55	0.00
	bm_t	0.15	1.44	0.04		bm_t	0.14	1.26	0.03

⁹ The concept is similar for db ($\phi=1.00$) and mdb ($\phi=0.87$), as well as be ($\phi=0.62$) and mbe ($\phi=0.55$).

¹⁰ Findings are available upon request.

mbm_t	0.41	1.46	0.13	mbm_t	0.30	1.05	0.07
be_t	0.62	6.37	0.30	be_t	0.63	3.59	0.27
mbe_t	0.84	7.63	0.41	mbe_t	0.77	3.77	0.32
db_t	-0.04	-0.93	0.03	db_t	-0.03	-0.55	0.01
mdb_t	0.12	0.77	0.01	mdb_t	0.07	0.36	0.00

Note: Standard errors are GMM corrected. Data is annual spanning the period 1926-2018.

Dividend growth variations are less identifiable by the ratios employed in this study. As evident in Table 4, bm and better yet mbm are capable to capture some predictive components at the medium and longest horizons, providing a maximum R^2 of 24% but still outcomes are limited compared to return forecasting. Our findings find great similarity to other studies that have employed various valuation ratios to predict dividend growth (see for example Ang and Bekaert, 2007; Cochrane, 2008; Chen, 2009). Possible explanations of this limited evidence may be that (a) dividends stand as a poor measure of true-relevant cash-flows since they are susceptible to manipulation, smoothing, censoring and even changes in firms' corporate financial policy (see argumentation in Chiang, 2008; Chen et al. 2009), (b) the positive correlation between expected dividend growth and expected returns may act as a deterrent to the forecasters' ability.

Furthermore, predicting earnings growth remains a challenge by all the forecasters employed except book-earnings (be) that performs astonishingly well despite its weak performance in returns and dividend growth forecasting. In fact, it manages to explain 46% of earnings growth variations at the medium horizon and produces a slope coefficient above 1, even though its capability slightly reduces in the longest h . These findings arouse interest for further research since no pivotal conclusions can be drawn on whether earnings growth is predictable after all.

Finally, we further examine the performance of the ratios in two sub-samples, that is the pre and post-1965 periods, so as to test their dynamics and compare outcomes to a more recent environment, including the recent economic turbulences from 2008 and onwards.

3.2 Evidence on sub-sampling

We proceed on examining our entire sample into two different but economically significant sub-periods, namely the pre and post-1965 periods, running similar in-sample forecasting regressions so as to further examine the ratios' forecastability. A similar approach is followed in Pontiff and Schall (1998) who support that there are structural differences in both sub-periods; the ability of bm to predict returns scatters away after 1960s and attribute such behavior in the data's very nature not being representative enough of the equities market as a whole.

Table 5: Sub-sampling: In-sample predictability of nominal returns for the pre-1965 sub-period.

		b	t(b)	R^2			b	t(b)	R^2
$r_t(3)$	bm_t	0.95	4.99	0.55	$r_t(5)$	bm_t	1.16	9.23	0.60
	mbm_t	1.23	6.52	0.67		mbm_t	1.54	13.80	0.78
	be_t	0.30	1.09	0.07		be_t	0.33	0.81	0.06
	mbe_t	0.38	1.33	0.10		mbe_t	0.46	1.08	0.10
	db_t	-0.07	-0.77	0.01		db_t	-0.21	-1.33	0.05
	mdb_t	1.19	3.30	0.46		mdb_t	1.46	4.93	0.50
$r_t(7)$	bm_t	1.01	8.90	0.47	$r_t(10)$	bm_t	0.53	6.76	0.42
	mbm_t	1.32	6.05	0.63		mbm_t	0.87	11.85	0.52
	be_t	0.09	0.32	0.01		be_t	0.37	1.53	0.07
	mbe_t	0.19	0.68	0.02					
	db_t	-0.32	-1.62	0.08					
	mdb_t	1.46	5.38	0.53					

Note: Standard errors are GMM corrected. Data is annual.

Table 6: Sub-sampling: In-sample predictability of nominal returns for the post-1965 sub-period.

		b	t(b)	R^2			b	t(b)	R^2
$r_t(3)$	bm_t	0.11	1.21	0.05	$r_t(5)$	bm_t	0.21	1.97	0.12
	mbm_t	0.19	1.45	0.07		mbm_t	0.37	3.77	0.17
	be_t	-0.03	-0.16	0.00		be_t	0.16	0.62	0.02
	mbe_t	-0.04	-0.16	0.00		mbe_t	0.21	0.81	0.03
	db_t	0.05	0.81	0.03		db_t	0.09	0.83	0.05
	mdb_t	0.40	12.73	0.25		mdb_t	0.63	9.53	0.38
$r_t(7)$	bm_t	0.33	2.75	0.24	$r_t(10)$	bm_t	0.53	6.76	0.42
	mbm_t	0.59	6.97	0.34		mbm_t	0.87	11.85	0.52
	be_t	0.24	0.84	0.04		be_t	0.37	1.53	0.07

mbe_t	0.32	1.23	0.05	mbe_t	0.44	2.26	0.07
db_t	0.12	0.80	0.07	db_t	0.26	1.74	0.20
mdb_t	0.89	11.62	0.60	mdb_t	1.18	16.03	0.77

Note: Standard errors are GMM corrected. Data is annual.

As shown in Table 5, we confirm the findings of Pontiff and Schall (1998). The conventional bm provides early on at $h=5$ an R^2 of 60%. The modified ratio though remains clearly superior regardless the horizon and manages to explain up to 78% at the 5-year return horizon. Also, mdb attains a maximum R^2 of 53% at the longest horizon, unlike the simple db whose performance is limited throughout all horizons.

Table 6 on the other hand, summarizes the outcomes for the post-1965 sample. There are mainly two critical observations to make; first, results are of lesser magnitude compared to the pre-1965 sample. This also finds reference to the findings by Pontiff and Schall (1998) who argue that the ratio's ability to predict returns is mainly related to the forecastability of book value to predict future cash flows. Second, the superiority of mbm still holds being in a position to explain 34% and 52% at the 7- and 10-year (medium-to-long and long) horizons ahead variations of the market, while the classical bm provides R^2 s of 24% and 42% equivalently¹¹. An interesting finding though is the predictive capacity of mdb which seems to perform better not only against its conventional counterpart but also against all other predictors, managing to attain an R^2 of 77% at the longest horizon.

One possible explanation regarding the forecastability of bm is that book values is a good proxy for future cash flows. The product of dividing a cash flow proxy by a current market price is a variable which is strongly correlated with future returns. This discount rate proxy affects firms' market capitalization which may fluctuate over time, regardless of rational or irrational factors. Consistent with this rationale, Pontiff and Schall (1998) construct a bm based on DJIA data and a bm based on S&P data and find that the latter is a better predictor of market returns, while also the S&P book-value is superior in predicting market cash flows. The relation between book values and cash flows may need further examination so as to help us comprehend the predictive capacity of the book-valued ratios to a greater extent.

3.3 Out-of-sample performance

As usual in return predictability studies apart from in-sample forecasts, econometricians evaluate the predictors' out-of-sample (oos) performance as well, that is the ability of the forecasting model to generalize on an independent test data set. We follow a straightforward concept by assuming that $L(y, \hat{v}(x)) = (y - \hat{v}(x))^2$ is the loss from a prediction $\hat{v}(x)$ for a target return y and a forecaster x on a training set. Our goal is the minimization of the so-called out-of-sample (or generalization error) which represents the expected loss over an independent sample. There is respectively an in-sample (or training) error which stands as the average loss within the training sample, but this is totally different to the generalization error. There is a key relationship between the two kinds of errors; the greater the in-sample error, then the less overfit the model is to the data set, and thus the greater it generalizes. If for instance, we consider Fama and French's (2002) preposition and allow for occasional breaks to the levels or the slope of a stationary process then we will receive increased slope coefficients and R^2 s in-sample but low oos R^2 s. That is mainly due to the weakening power of unit root tests to identify stationary processes with breaks in comparison to the non-stationary ones (as argued first in Perron, 1989). After careful consideration of these effects, we suggest a modified technique (as first shown in Polimenis and Neokosmidis, 2016) which provides significant oos predictive gains.

We proceed on evaluating the forecasting capability of the conventional and our modified ratios out-of-sample (oos) on nominal returns. We use the well-established Campbell and Thompson (2008) technique who estimate an R^2_{oos} statistic by comparing the out-of-sample performance of a selected predictor with return forecastability against a simple forecast benchmark that is based on the simple average of past returns as a forecast. The proposed out-of-sample coefficient of determination is computed via the formula:

$$R^2_{oos} = 1 - [\sum_t (r_t(h) - \hat{r}_t(h))^2 / \sum_t (r_t(h) - \bar{r}_t(h))^2] \quad (15)$$

where $\hat{r}_t(h)$ stands as the h -years out predicted return using information at time t based on a predictive regression, while $\bar{r}_t(h)$ is the historical average h -year return. We include forecasts at $h=5, 7, 10$ -year horizons on returns. The first step is to divide the sample into an estimation and an evaluation period. In our case the first 15-year period (1926-1941) is considered the minimal estimation period since the quantity of the data must be enough to increase reliability of OLS estimators. The rest of the sample (till 2018) constitutes the evaluation period for which enough data is also needed to ensure reliability of out-of-sample estimates (see the discussion in Goyal and Welch, 2008; Campbell and Thompson, 2008).

In order to estimate the modified versions of the simple ratios it is critical to firstly calculate the true population coefficient β in eq. (1) (and similarly γ in eq. (2) and δ in eq. (4). This could manifest as a re-estimation of the cointegration coefficients between b and m (b and e , and d and b equivalently) on a recursive basis, which implies using data up to a specific point t in time. We need to consider though that this approach carries great sampling errors and eliminates the modified ratios' forecastability. We believe that this happens because forecasting regressions

¹¹ We have also sub-sampled excess and real returns at similar horizons with no significant changes in the outcomes.

are run against a proxy mbm_{rec} (mbe_{rec} and mdb_{rec} respectively) instead of the true population coefficient β (or γ or δ).

In an attempt to moderate the aforementioned effect, we proceed on estimating both the recursive but also the full sample oos forecasting performance of the examined ratios (simple and modified) following the concept of Polimenis and Neokosmidis (2016, 2019). We mainly report though the oos R^2 s produced by the suggested approach (that is when the cointegrating coefficients are calculated on the entire sample) since the predictive benefits are more robust. However as discussed in Lettau and Ludvigson (2001, 2005) and Polimenis and Neokosmidis (2016, 2019) there is a look-ahead concern when we estimate mbm_{full} (mbe_{full} and mdb_{full} respectively)¹². When we perform analysis in sub-samples, we do not use all available information in estimating parameters thus, we do not “see” the entire predictive capacity as measured with in-sample tests. Yet, the most appropriate approach when modifying simple ratios is to employ the full sample since enough data is needed to ensure reliability of the cointegrating coefficients (see also the case of the cay and cdy variables by Lettau and Ludvigson, 2001, 2005 and the case of the dpe variable by Garrett and Priestley, 2012).

Table 7: Out-of-sample (oos) forecasting.

Returns	$r_t(3)$	$r_t(5)$	$r_t(7)$	$r_t(10)$
bm_t	-1.163	-1.025	-0.427	-0.136
mbm_t	-0.799	-0.656	-0.114	0.156
be_t	-0.190	-0.075	-0.044	-0.089
mbe_t	-0.156	-0.037	-0.010	-0.034
db_t	-0.188	-0.717	-1.310	-1.609
mdb_t	-0.160	0.078	0.473	0.710

Note: We present OOS forecasts for the conventional and modified book-valued ratios. Data is annual for the period 1926-2018.

As mentioned earlier, Table 7 summarizes the results of out-of-sample estimations on nominal returns by all included forecasters based on the full-sample approach. The findings show that (a) the modified ratios provide out-of-sample (oos) improvements against the conventional ones as horizon is extended, (b) mbm proves superior even oos against bm , and (c) the modified db ratio surpasses all ratios included in this study, managing to attain an R^2_{oos} of 71% for $h=10$.

An investor who has seen enough of the entire sample to infer to the cointegration beta with relative confidence will enhance his forecastability for the 7- and 10-year returns by an astonishing R^2_{oos} of 47% and 71% by employing mdb . Unlike mdb though, it is only at the longest horizon that even mbm is able to generate substantial predictability at 15,3% while results are poor for all the other horizons. To illustrate the oos superiority of mdb against mbm , we plot the R^2_{oos} of the two ratios throughout all forecasting horizons in Figure 2.

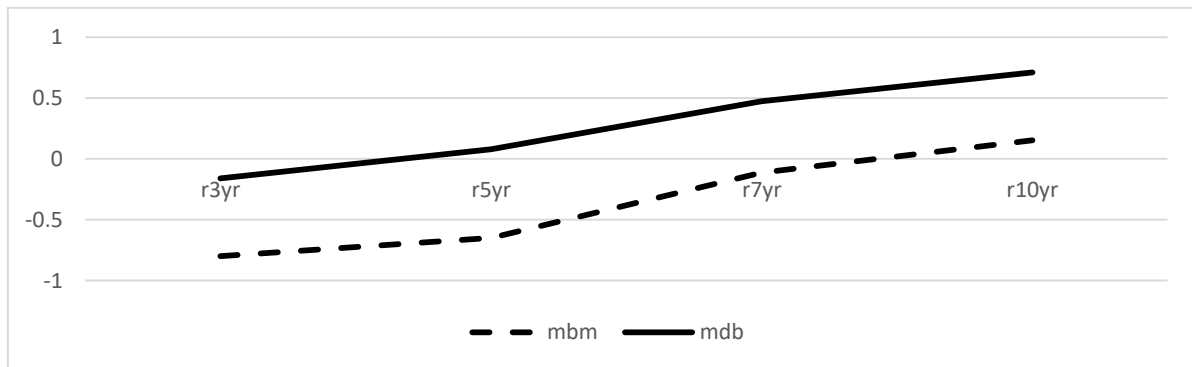


Figure 2: Out-of-sample (oos) forecasting in horizons for annual nominal returns.

Note: The figure shows the evolution of R^2 as we extend the horizon of our forecasts for the modified bm and the modified dividend-book ratios. Data is annual spanning the period 1926-2018.

Additionally, it is evident that all conventional ratios cannot generalize well oos regardless the examined horizon which is in line with existing findings in empirical literature. The issue in most studies is that even though predictability evidence is retrieved in-sample by several valuation ratios (some may be more powerful forecasters than others, but still there is enough predictive ability observed throughout finance literature), it remains trickier to identify models and ratios with similar forecastability out-of-sample (see for instance, Goyal and Welch, 2008; Lettau and Ludvigson, 2005; Cochrane, 2008). Consequently, all the negative R^2_{oos} we retrieve not only by the conventional ratios but also by be and its modified counterpart, confirm previous research and can be mainly interpreted as the inability of these ratios to outperform the simplistic forecast benchmark in all forecasting horizons¹³.

¹² This concern addresses the issue of the econometrician seeing enough of the historical data to explore the utter forecasting ability of the modified ratios to his advantage.

¹³ We have also run similar forecasts on excess and real returns without any critical change in the outcomes.

4. Conclusion

This paper examines the forecasting ability of the conventional book-market ratio on S&P 500 high quality annual data covering the period 1926–2018. Our primary focus lies on (a) testing the forecasting performance of book-market (bm) ratio and modify it accordingly based on the long-run equilibrium relationship between book and market values; (b) constructing two new ratios mixing dividend and earnings series with book values and (c) proposing that the modified ratios which basically stand as the stationary trend deviations of the simple ratios, manage to provide substantial forecasting improvements.

The main findings of our study are the following. First, we retrieve long-run equilibrium relationships not only among book and market values, but also between book and earnings, and dividends and book values. Second, the modified book-market (mbm) has a more enhanced nominal return in-sample fit over the conventional bm while third, the modified dividends-book (mdb) ratio performs even better in-sample, managing to explain 59% of total return variations for $h=10$. Fourth, the book-earnings (be) ratio may not have as strong in-sample fit as the rest but yet, bears fruitful evidence in earnings growth forecasting when both bm and db present poor results. Also, sub-sampling allows us to test the predictive capacity of the examined ratios in a more recent environment; we conclude that bm is a more capable predictor in the pre-1965 sample reconfirming findings of previous studies.

Regarding the ratios' out-of-sample (oos) performance, the modified bm manages to surpass the simplistic forecast benchmark at the longest horizon while bm still struggles throughout all forecasting horizons. The impressive finding is observed at the case of db that attains an astonishing R^2_{oos} of 47% and 71% at 7- and 10-year returns ahead. However, book-earnings along with its modified counterpart do not generalize well on an independent sample thus further research is required so as to comprehend this extra complexity of the modified approach for earnings.

Overall, we provide valid evidence that the simple bm ratio has impressive forecastability that should not be overlooked by either the dividend or earnings yields. By associating book values to dividends and earnings we manage to increase predictive benefits and provide fresh evidence on one of the strongest indexes in the market. We strongly believe that our work could prove beneficial to investors, portfolio and risk managers, financial analysts, as well as scholars and other researchers on the field. We particularly address one of the most crucial questions in empirical finance of what makes returns predictable and we are confident that we can help practitioners face most of the constant challenges related to these issues. Further research is required so as to solve the earnings puzzle in terms of forecasting along with the necessity to understand the economical sources behind non-stationarity in valuation ratios.

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Appendix

As described in the main text we follow several steps before retrieving evidence of long-run equilibrium relationships and constructing the modified version of the simple ratios. The most important reason why instead of employing the Engle-Granger (EG) method, we focus on the Johansen technique (1988, 1995a) is because the latter manages to eliminate two limitations of the first. In particular, EG approach includes two steps; the 1st regression automatically transfers errors in the residuals to the 2nd regression which tests for unit roots. Also, the estimated (but not observe) residuals require different tables of critical values for standard unit root tests.

Since the analysis of our methodology on the main text focuses on the $[b \ m]$ vector, here we describe the similar steps we follow for the $[b \ e]$ and $[d \ b]$. The notation we follow assumes $w_t = [b_t \ e_t]'$ and $z_t = [d_t \ b_t]'$ as the vectors of logged book (b) and earnings (e), and log dividends (d) and book (b) values. In order to test for stationarity of the conventional be and db ratios we impose a restriction $c = [1 \ -\beta] = [1 \ -1]$ on the Johansen estimated vectors. In order to specify the most appropriate lag length in each vector, we commence by calculating a VAR model in levels with the highest initial order autoregressive coefficients. We assume a maximum order of 12 lags and conclude based on HQ criterion that we should use 8 lags for VAR (thus, 7 lags for VECM) for vector w_t and 4 lags for VAR (thus, 3 lags for VECM) for z_t .

Table A1: VECM results between book and market values.

	α	c_t
Δb_t	-0.007024 (0.00327)	0.004366 (0.00114) R^2 0.004140
Δm_t	0.008470 (0.00461)	0.004740 (0.00161) R^2 0.003029

Note: The table presents the outcomes from the VECM estimation between the book (b) and market (m) values using the multivariate Johansen procedure. Data is annual for the period 1926-2018. (*) and (**) denote significance at the 5% and 1% rejection level respectively.

Table A2: VECM results between dividends and earnings.

	α	$\Delta w(t-1)$	$\Delta w(t-2)$	$\Delta w(t-3)$	$\Delta w(t-4)$	$\Delta w(t-5)$	$\Delta w(t-6)$	$\Delta w(t-7)$	c_t
Δd_t	- 0.009319 (0.00370)	- 0.029438 (0.03025)	- 0.019585 (0.02855)	0.044036 (0.02781)	- 0.029255 (0.02755)	- 0.015114 (0.02665)	0.043723 (0.02645)	0.007376 (0.02643)	0.003922 (0.00106) R^2 0.119241
Δe_t	0.010145 (0.00284)	0.049170 (0.03750)	0.321973 (0.04543)	0.0229817 (0.04673)	0.014454 (0.04655)	0.637651 (0.04623)	0.480253 (0.04999)	0.019521 (0.4300)	0.236592 (0.00082) R^2 0.243612
		0.680651 (0.02881)	0.110571 (0.03491)	0.080940 (0.03591)	- (0.03577)	0.025429 (0.03552)	0.150439 (0.03841)	- (0.03305)	0.001445 0.643549

Note: The table presents the outcomes from the VECM estimation between the 12-month summed-up dividends (d) and 12-month summed-up earnings (e) using the multivariate Johansen procedure. Data is annual for the period 1926-2018. (*) and (**) denote significance at the 5% and 1% rejection level respectively.

Table A3: VECM results between dividends and book values.

	α	$\Delta w(t-1)$	$\Delta w(t-2)$	$\Delta w(t-3)$	c_t
Δd_t	-0.006728 (0.00225)	0.126819 (0.02820)	0.104528 (0.02830)	0.367696 (0.02823)	-0.000405 (0.00067) R^2 0.205155
Δb_t	0.000124 (0.00394)	-0.011495 (0.01725)	-0.000788 (0.01726)	0.026777 (0.01724)	0.004438 (0.00117) R^2 0.002797

Note: The table presents the outcomes from the VECM estimation between the 12-month summed-up dividends (d) and book values (b) using the multivariate Johansen procedure. Data is annual for the period 1926-2018. (*) and (**) denote significance at the 5% and 1% rejection level respectively.

Even though we first test for a cointegration relationship that contains only a constant and has linear trends (also known as deterministic cointegration), we retrieve evidence that there is a long-run equilibrium relationship of this sort only in the vector w_t , while in the case of z_t there is no deterministic trend (similar is the case for $[b \ m]$).

Similarly, to Panel B of Table 2a, the respective Panels in Tables 2b and 2c show the results of examining the restriction that the vector $[1 \ -1]$ spans the cointegration space based on the Johansen technique on $[b \ e]$ and $[d \ b]$ which is also rejected at 5% critical level. This constitutes an even more robust indication that bm and its by-products (our be and db) behave in a non-stationary manner and thus, deal with the lower power of unit root tests against highly persistent alternatives.

ARDL Analysis of Remittance and Per Capita Growth Nexus in Oil Dependent Economy: The Nigeria's Experience

Mathew Ekundayo Rotimi¹, Michelle Doorasamy², Udi Joshua³, Grace Gift Rotimi⁴, Confort Omolayo Rotimi⁵, Gabriel Samuel⁶, Gbenga Adeyemi⁷, Ayodele Solomon Alemayo⁸, Alfred Kimea⁹

¹Department of Economics, Federal University Lokoja, Kogi State, Nigeria, mathew.rotimi@fulokoja.edu.ng, <https://orcid.org/0000-0002-0938-9791>

²School of Accounting, Economics and Finance, College of Law and Management Studies, University of KwaZulu Natal, Durban, South Africa, doorasamym@ukzn.ac.za, <https://orcid.org/0000-0001-9320-3461>

³Department of Economics, Federal University Lokoja, Kogi State, Nigeria, <https://orcid.org/0000-0002-7862-0547>

⁴Department of Business Administration, Prince Abubakar University, Anyigba, Nigeria, gracegift29@gmail.com, <https://orcid.org/0000-0002-9775-4903>

⁵Department of Economics, Federal University Lokoja, Kogi State, Nigeria, <https://orcid.org/0000-0002-5319-7930>

⁶Department of Accounting, Ibrahim Babangida University, Lapai, Niger State, Nigeria, gabrielsamuel52@gmail.com, <https://orcid.org/0000-0003-1474-6498>

⁷Wesley University, Ondo, ondo State, Nigeria, gbenga.adeyemi@wesleyuni.edu.ng, <https://orcid.org/0000-0001-6843-3054>

⁸College of Education (Technical), Gbeleko, Kabba, Kogi State, Nigeria, <https://orcid.org/0000-0002-9436-8580>

⁹Institute of Tax Administration, Tanzania, kimeahild@yahoo.com

ARTICLE INFO	ABSTRACT
Article History	Purpose:
Received 26 September 2022; Accepted 13 January 2023	Remittance is essential to economic wellbeing. Realising this fact, this study examined, within the optimist theoretical framework, whether international remittances significantly impact per capita economic growth in Nigeria.
<i>JEL Classifications</i> O47, F43, F24, C22	Design/methodology/approach:
	Employing annual time series data spanning 1980-2020, the study adopted the Pesaran, Shin, and Smith ARDL bounds estimating model to examine the type of relationships between remittances and Nigeria's per capita growth.
	Finding:
	The study reveals a statistically significant positive nexus in the long-run and short-run among the variables. Specifically, it found that higher remittances inflow enhances per capita growth both in long-run and short-run in Nigeria. Furthermore, the study found that remittances are sources of external financing and eventually, it is a means to economic growth and also may help to fill fiscal deficit gap.
	Research limitations/implications:
	This study recommends that government should, through sound policy option, encourage remittances influx. This could be realised by creating viable relationship among international communities that largely account for remittance inflow into Nigeria. It further suggests a prudent and optimal management of remittances inflow through the appropriate monetary authority. This may include formulating policy that will ease remittance inflow and remove unnecessary barriers to inflow of remittances.
Keywords: Remittances, Per Capita, ARDL, Growth, Oil Dependent, Nigeria	Originality/value:
	The study contributes to literature by examining whether international remittances within the optimist theoretical framework significantly impact per capita economic growth (PCEG) in Nigeria.

¹Corresponding Author: Mathew Ekundayo Rotimi
Email: mathew.rotimi@fulokoja.edu.ng

1. Introduction

Remittance may simply be referred to as earnings by migrant workers into their home nation. In the literature, workers' remittances are defined by the International Monetary Fund (IMF) as amount of monetary transfers sent by those workers who have been resident abroad for over a year to their country homes, and they are documented in various segments of balance of payments (Sutradhar, 2020). It is believed that remittances impact economic activity, especially in the developing nations. For instance, Sutradhar (2020) argued that, for developing nations, remittances constitute an increasingly significant mechanism for the resource transfers from the first world nations to the developing nations, and workers' remittances, in term of volume, are the 2nd-largest source, next to foreign direct investment (FDI) and external funding (Russell 1986; Sander and Maimbo, 2005; Buch and Kuckulenz, 2010; Sutradhar, 2020; Karagoz, 2009).

Buttressing the claim that remittances may impact economic growth, various studies have argued that remittances, over the current decades, are veritable, consistent, and safe source of foreign/external finances inflow among the developing countries (Rao and Hassan, 2011; Kudaisi et al., 2021; Adenusi, 2011; Giuliano and Ruiz-Arranz, 2009). Consequently, while a few researches have attempted to consider the economic growth and remittances nexus in the short-run, the analysis of the remittances and economic growth nexus in the long-run has not received much attention (Tolcha and Rao, 2016; Mwangi and Mwenda, 2015). Specifically, causal linkages in remittances and economic growth have received negligible attention in the developing countries, leaving the doubt of whether remittances would, in the long-run, be impactful to economic growth (Jouini et al., 2021; Bettin et al., 2014). Furthermore, economic growth nexus and remittances, in the case of oil dependent nation like Nigeria, having a remarkable portion of its population abroad, may differ from earlier findings. Buttressing this assertion, Wadood and Hossain (2015) allude that the issue of the remittances and economic growth nexus of recipient economy has remained contentious. The perceived impact of international remittances on economic growth of recipient economy has remained debateable among academics. This results to sluggishness in decision making and imbalances in balance of payment. The conflicting academics are the optimists, pessimists, and the liberals. The optimists avow that the overseas remittance has positive influence on the economic growth of the recipient economy, thus, leading to increase in investments and development of human capital (Wadood and Hossain, 2015; Garcia-Fuentes et al., 2009; Mwangi and Mwenda, 2015).

The pessimists, on the other hand, claim that overseas remittances unfavourably influence economic growth of the recipient economy exerted by poor orientation and inflationary pressure which emanate from insufficient labour supply (Karagoz, 2009; Chami et al., 2005; Davis and Carr, 2010). The liberal academics are indifferent. They contend that overseas remittances don't impact economic growth of the recipient economy (Shimul, 2013). Following the inconclusiveness on the issues of overseas remittances and economic growth nexus, there is a need for empirical investigation which this study undertakes. This study primarily examines overseas remittances and on per capita growth nexus.

Several studies have examined economic growth and remittances nexus in Nigeria. Some of these studies have shown that remittances significantly and statistically impact economic growth both in long-run as well as short-run (Oshota and Badejo, 2014). Similarly, study like Adarkwa (2015) employed OLS regression model and the study revealed that remittances impact economic growth. These findings contravene Kumar (2011) who found that remittances have significant negative and positive impact on economic growth. Furthermore, the study argued that the negative impact of remittances in the short-run is associated with the fact that beneficiaries of remittances keep money idle by saving it in short-run, while the positive effect in long-run is connected to the fact that the money saved is economically employed in the long-run to bankroll capital projects.

Earlier research on economic growth and remittances nexus have provided evidence on the connection between remittances and various economic growth indicators such as GDP and RGDP. The favourable and unfavourable pull and push effect of remittances on economic growth motivates this study. While the earlier studies on remittances have emphasized the motive and advantages of worker remittances, the manner in which remittances per capital income remains inconclusive in the literature. This could have offered a better explanation on how remittances could impact economic growth. In addition, such explanations could have examined, with regards to the long-run and short-run, the effect of remittances on per capita income. The study also seeks to determine the causal linkages between per capital income and remittances. This study is informed by the literature which shows that previous study on remittances-economic growth nexus have largely employed panel data to examine emerging countries, thus, making it problematic to consider country particular issues (Sutradhar, 2020; Fayissa and Nsiah, 2010; Feeny et al., 2014; Nyeadu et al., 2014). This study, among other things, contributes to the literature by examining whether international remittances, within the optimist theoretical framework, significantly impact PCEG in Nigeria.

The remainder of the study, following the introduction, is arranged as follow. Section 2 gives overview of remittances. Section 3 reviews literature, section 4 presents the methodology to the study, section 5 discusses findings from the study and section 6 concludes the study and offers relevant recommendations.

2. Overview of remittances

This section gives a general overview of remittances and this is cascaded down to the Nigerian context.

2.1 Remittances: A Global Overview

Globally, about twenty percent of the primary research involve an interaction stretch between remittances and one independent variable or the other (Cazachevici et al., 2020; Konte, 2018; Kratou and Gazdar, 2016; Tsaurai, 2015). The study observes that development in the financial sector is the commonest taming factor that is used in interaction terms. Supporting this claim, Mundaca (2009) notes that remittance has a positive long-run impact on economic growth, but financial inclusions could further improve the positive connection. Contrary to this view, Mohamed and Sidiropoulos (2010) opine that remittance positively impact economic growth with and without financial development and interacting remittances. Nonetheless, Bettin and Zazzaro (2012) reveal that remittances would exhibit a positive impact on economic growth in nations that have an efficient internal banking sector that could serve as an efficient intermediary in channelling remittances to growth-improving projects.

Clemens and McKenzie (2018) carried out study on selected global developing countries and note that nations where remittances constitute a large proportion of their GDP haven't experienced remarkably higher growth over a twenty-year period than nations which receive much lesser remittances. This study revealed that nations vary from one another in a numerous of characteristics, and this has necessitated various empirical works to either implicitly or explicitly ask if variations in remittances could lead to economic growth. The study also observes that striking growth in projected remittances hasn't been supplemented by palpable variations in economic growth for the nations that receive them. Buttredding this submission, Imad (2017) shows that while it is evident that institutions contribute to economic growth, evidence on direct relation on economic growth and remittances nexus is not well documented in the literature. Although, Ball et al., 2013 advocates that remittances spur inflation and this is a component of growth in nominal GDP. While several studies have attempted to understand the economic growth and remittances nexus, the relationship between the duo in developing oil exporting countries, assumed to be attractive to significant remittances inflow, appears to be unattended to. Hence, the need for this study:

Cazachevici et al. (2020) opine that the remittance of expatriate workers constitutes a vital source of finance to middle- and low-income nations, yet there is no agreement, that has emerged on remittances and economic growth nexus. In their study for instance, where a survey of 538 samples was reported in ninety-five studies, the study reveals that nearly 40 percent of the studies has a positive impact, 40 percent reveals no impact, and 20 percent reveals a negative impact. These findings indicate a bias in favour of positive impacts. Despite needful corrections to the bias, employing recently developed methods, the findings reveal that the mean impact remains positive but economically little. Yet, their findings unveiled remarkable differences in the regions. For instance, they found that remittance is growth-improving in the Asian nations but otherwise in Africa.

2.2 Remittances in Nigeria: An Overview

Nigeria is a middle-income nation with expanding financial and manufacturing sectors. It ranks the largest economy in Africa and 27th largest globally, in terms of nominal gross domestic products. Oil resources constitute the main stay of the Nigerian economy. The sector contributes about 60% of its revenues. Oil alone contributes nearly 9% of the GDP. It produces only nearly 2.7% of the global oil supply (OPEC, 2020). Though, the petroleum sector is vital as government revenues, yet, Nigeria is one of the countries regarded as the largest recipient amongst remittances recipient nations. Over time the international remittance to Nigeria has steadily improved and the remittances have become a major share of the financial inflows to the country (see Figure 1).

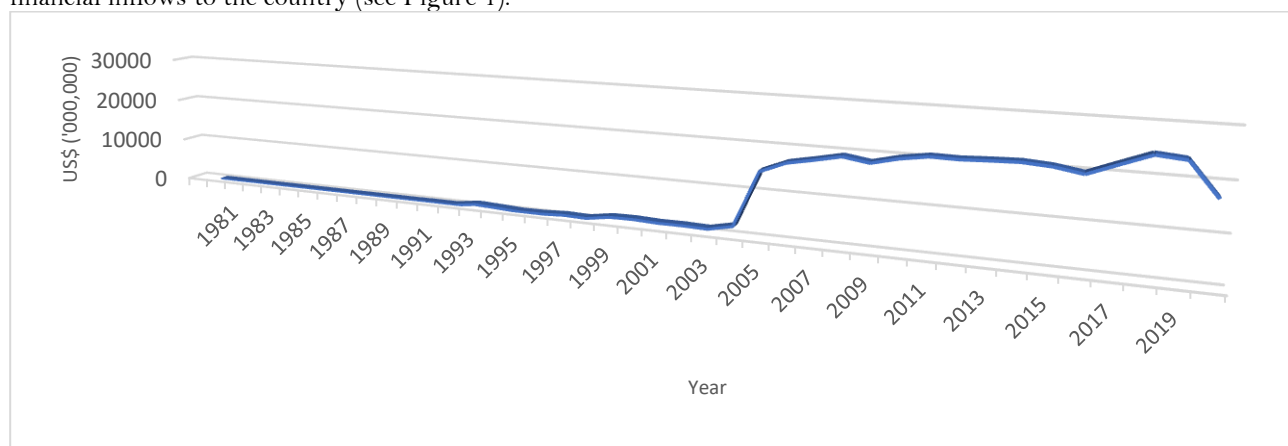


Figure 1: Remittance inflow to Nigeria

Source: computed by the authors from the data obtained from World Bank (2020).

In Africa, Nigeria is the 5th largest recipient of international remittances followed by Senegal. Nigeria received the sum of US\$21 billion remittances in 2013. Undeniably, Nigeria emerged the leading recipient of remittances in SSA in 1990, and from 2006, it has been the leading recipient of remittances in Africa, including North Africa (Laniran and Adeniyi, 2015; Nyamongo et al., 2012). The rise in the international remittances to Nigeria may not be unconnected with increase in the population of her resident living abroad. For instance, at \$23.8 billion, Nigeria received nearly half of the remittances sent to sub-Saharan Africa (SSA) in 2019. South Sudan, however, is estimated to have the highest remittance flow as a percentage of GDP in 2019 (34.4%). In 2020, it received about \$16.9 billion in international remittance flows compared to \$23.4 billion in 2019 (World Bank, 2020).

According to Constantinescu and Schiff (2014), rise in international migration leading to increase in remittances is a vital factor propelling the growth of global remittances, Nigeria inclusive. Nigeria has the largest population in Africa, and seventh largest in the world, having over 200 million people (World Bank, 2020). The population of the country accounts for approximately 25% of the sub-Sahara African population. Following World Bank (2020), the projected remittances receipt for Nigeria for 2014 was US\$22.3 billion. This constitutes an amount which is US\$14.4 billion greater than the collective amount received by the other leading ten largest receivers of remittances in sub-Saharan Africa (SSA). Senegal is next to Nigeria among the SSA nations, receiving a total of US\$1.7 billion international remittance, about 7.6% of the overall remittances to Nigeria. This assumed position by Nigerian stands Nigeria out in the literature discourse among the sub-Saharan African nations.

Between 1980 and 2004, remittance averagely constituted about 0.67% of the GDP, peaked at 8.31% in 2005 and began to decline steadily. Though, the contribution of remittance to GDP from 2007 to 2020 averaging 5.27% is remarkable compared with 1980-2004. The steadily decline in the proportion of remittance to GDP may have its impact on economic. The extent of this could be empirically verified.

3. Review of Literature

This section presents the literature review to the study. There are several debates on international remittances and economic growth nexus among academics, researchers, and policymakers. These debates have continued to be equivocal within the context of both theoretical and empirical views. Theoretical views on remittances differ, but the conformist insight submits that a large magnitude of remittances in a country, especially the developing country accounts for economic growth of the country (Garcia-Fuentes et al., 2009; Mwangi and Mwenda, 2015; Wadood and Hossain, 2015). A few empirical studies have divergent to the conventional view. For instance, Kumar (2011) argued that remittances have significant positive and negative effect on economic growth in the long-run and short-run respectively.

3.1. Theoretical Review

Several theories underpin remittances and economic growth relationship. The remarkable preoccupation of these theories is whether remittances impact economic growth, how and to what extent it affects it. Therefore, these theories advocate those remittances speed up economic growth. From viewpoint, a few people would disagree with the fact that remittance has positive impact on the economy of the recipient countries. Among various theories on remittances and economic growth nexus, this study considers the optimist (*Developmentalist/Neo-Classical*), and *Structuralist/Dependency* in De Haas (2010) and Adarkwa (2015) to ascertain how remittances impact economic growth. They are suitable to examine remittance-economic growth nexus.

3.1.1 The Optimist View

This is also known as developmentalist or Neo-Classical view. It evolved in the fifties with the supposition that, through transfer of capital, adoption of western culture and industrialisation, developing nations could accelerate their developmental process (Adarkwa, 2015). According to the study, this period experienced underdevelopment and the underdevelopment was ascribed to internal factors in developing nations. Accordingly, the study avows that if developing nations were willing to develop, the developing nations needed to refrain from their norms as well as cultural and traditional way of doing things and embrace modernity from the western world (Coetzee, 2001).

Furthermore, the prominent scholars of this theory among which include and Massey (2020), Beijer, (1970) as cited in Olayungbo and Quadri (2019). Todaro (1969) argue that migration would lead to transmission of capital investment through remittances. The procedure exposes primitive/traditional cultures to a more liberal, democratic and rational ideas such that will aid development (De Haas, 2010). Accordingly, labour migrations are perceived as core portions of transformation and it is assumed that the impacts of migration on growth and development could be viewed via the inflow of remittances which can help to enhance incomes and productivity (Massey, 2020). In view of this, migrants' remittances

are vital because they cause variation in household incomes, stimulate innovations and investments, and largely aid the overall economy of the migrants' nation of origin in its economic take-off (Olayungbo and Quadri (2019).

3.1.2. Pessimists Views

Contrary to the optimist view, the pessimists view, also referred to as structure and dependency theory, advocates that remittances and migration lead to underdevelopment in the country home of migrants (Adarkwa, 2015). Prominent scholars linked with this theory are Binford (2003) and Rubenstein (1992). Furthermore, the theory argues that different from making receivers of remittances reliant on the senders, international remittances cause the receiving nations to be dependent on the countries sending remittances (Adarkwa, 2015; Binford, 2003). Buttressing this claim, De Haas (2010) avow that migration is a conduit through which human capacities of communities are drained, hence, leading to passive development, in addition to making the receiving countries remittance-dependent. Consequently, Lipton (1980) cited in Nyasha (2019) and Oluwafemi and Ayandibu (2014) concludes that remittances would lead to imbalances rather than promoting economic growth Lipton (1980), because in most cases, remittances received are usually used to procure conspicuous items like houses, vehicles, fashions rather than exploring investment opportunities. This may further worsen income inequalities among individuals in the remittance receiving countries between remittance receiving households and those that don't receive any (Oluwafemi and Ayandibu, 2014; De Haas, 2010), hence, leading to inflation in the remittent recipient countries.

3.2 Previous studies

The literature reveals that various researches has studied economic growth and remittances nexus among the recipient nations. Specifically, the studies investigated the impact of remittances on economic growth among the receiving countries.

The literature reveals that remittances largely and positively contribute to household welbeign in the receiving counties (Nyeadi et al., 2014; Adams 2010). For Instance, Lopez et al. (2007) find that, in Latin American and Caribbean, remittances have remarkably assisted to lessen poverty, inequality, and have improved economic growth. They established that remittances have become sources of income among the poor individuals in developing nations. Nonetheless, the effect of remittances on economic growth, it is still contentious. A few studies have argued that causal relationship between growth and remittances might not be completely fathomable employing instrumental variables while the remittances effect on some economic variables is not evident in the short term (Nyeadi et al., 2014; Adams 2010).

Sutradhar (2020) employed balanced panel data, for four South Asian developing nations, covering 1977-2016 to examine the effect of workers' remittances on economic growth. The study shows a negative link in economic growth and remittances in Sri Lanka, Pakistan, Bangladesh, but a positive link between economic growth and remittances in India. In addition, it reveals a joint negative but significant economic growth and remittances nexus in four countries.

Oshota and Badejo (2014) investigate remittances relationship with economic growth in Nigeria. Employing an Error Correction Model on data covering 1981-2011, the study finds that remittance has positive effect on economic growth in Nigeria. Furthermore, the study reveals that a 1 per cent increase in remittances would lead to a 0.19% increase in real GDP in the long-run, but a significant negative relationship with the real GDP in short-run. Jebran et al., (2016) investigate the impact of remittances on per capita economic growth in Pakistan for 1976 to 2013. Employing Auto Regressive Distributed Lag (ARDL) Bounds testing model, the study investigates both long and short-runs liaison of remittances with per capita economic growth. The study reveals a statistically significant positive long-run and short-run impact of remittances on per capita economic growth.

4. Methodology

The methodology to the study is discussed in this section. This study examines the effect of international remittances on per capita growth in Nigeria. Unambiguously, the study analyses short-run and long-run nexus in remittances and per capita growth in Nigeria, investigating whether remittances in the long-run could impact per capita growth in Nigeria. To appropriately model the data employed in this study to extract both the short-run and long-run relationships in accomplishing the objective of the study, the study among other things considers unit roots test and cointegration relationship with the data to choose a suitable methodology. This procedure is in line with Kutu and Ngalawa (2016) and Giles (2013) who enumerated the four situations below which are involved in deciding the methodology that is appropriate for a data set:

- i. Ordinary Least Square (OLS) applies, if variables are stationary at I (0).
- ii. Vector auto regressive (VAR) applies, if variables are stationary at I (1), but they are not cointegrated.

- iii. When variables cointegrated and also integrated of the same order, two types of regression models suggested are OLS regression model using the variable at levels, which provides the long-run equilibrium association between the variables and an Error Correction Model (ECM).
- iv. When some variables are stationary at $I(1)$, and $I(0)$, resulting to ambiguity when compared with (i)-(iii) above, ARDL is suggested

This study employs annual data which is line with earlier studies (Kudaisi et al., 2021; Bettin et al., 2014 ; Jouini et al., 2021; Tolcha and Rao, 2016; Mwangi and Mwenda, 2015). The data covers 1990-2021. The choice of data points is informed by data availability and how recent is the data. Furthermore, the decision for variables is formed on the bases of theory and earlier studies (Kudaisi et al., 2021; Bettin et al., 2014).

4.1 Data and data sources

Consequently, this study used are per capital growth (ζ), international remittance (IREM), exchange rates (ER), oil price (OP), and investment (INV). They have been obtained from the World Development Indicators (WDI) and statistical Bulletin of the Central Bank of Nigeria. Theory and previous study informed this choice.

4.2 Estimating Technique

This study adopted ARDL estimating technique, following Pesaran et al., 2001). It evaluates whether there is shortrun and longrun relationship between remittances and per capita growth in Nigeria. Furthering the choice for this technique, varying from the fact that it is appropriate for combining the $I(1)$ and $I(0)$ series which implies that variables that are integrated of $I(1)$ and $I(0)$ can be estimated in one regression; ARDL model can mutually cointegrate variables, ignoring their order of integration but not order $I(2)$ (Katircioglu, 2009). It makes ARDL model a more superior technique to other techniques used to investigate shortrun and longrun nexus. Furthermore, beside the fact that variables employed in an ARDL model could be assigned dissimilar lags, it is appropriate for both large and small sample sizes (Giles, 2013). The ARDL could synchronously estimate the long-run as well as shortrun parameters (Shin et al., 2014), and it contains a single-equation structure, thus, making it easy to apply and interpret (Giles, 2013).

4.3 Unit Root Tests

Bornhorst and Baum (2006) emphasises unit root tests. The study argues that the characteristics of the variables should be examined before they are used to conduct an ARDL analysis to avoid wrong specification of a model, which subsequently may result into loss of vital information about the data sets, and by extension a misleading value of R-square, F and t-statistics, and spurious results (Hamid and Shabri, 2017). A unit root test helps to produce consistent parameter estimates notwithstanding whether the time series are integrated or not, making it produce a more robust result. Following Bornhorst and Baum (2006) the study, used Augmented Dickey Fuller, Dickey Fuller, and Philip-Perron to test for stationarity of the variables. The choice of the multiple criteria to test for stationarity is dictated by the need to authenticate the consistency, reliability and validity of results (Frimpong and Oteng-Abayie, 2006).

4.4 Lag length

To select a suitable lag length, this study follows the conventional criteria available in the literature which states that lag with least criterion be given consideration (Lutkepohl, 2006). To permit adjustments in the model therefore, and achieve reliable and well-behaved residuals, lag order is chosen using robust criteria consisting AIC, HQIC, FPE, and SIC.

4.5 Diagnostic Tests

Alquist and Kilian (2010) argue favourably on the need to conduct basic diagnostic tests to investigate the consistency of the ARDL. Consequently, this study conducts diagnostic tests, heteroscedasticity, serial correlation, stability and normality tests to authenticate the appropriateness, reliability and robustness of the model.

Accordingly, this research work tests for both the null and alternative hypotheses of heteroscedasticity, serial correlation, and and normality are hypothesized as:

Null Hypothesis: $H_0: \xi = 0$, there is no heteroskedasticity, no serial correlation; and residuals are normally distributed.

Alternative Hypothesis: $H_0: \xi \neq 0$, there is heteroskedasticity, serial correlation; and residuals are not normally distributed.

As the name implies, the stability test examines the stability if the model. The stability test is rooted on the recursive chow test. This submits that for a model to be reliable, there must be stability over time. The stability test uses the graphical CUSUM to decide stability of the model. The benchmark specifications of hypotheses are as follow:

Null Hypothesis $H_0: \delta = 0$, the model is stable
 Alternate Hypothesis $H_0: \delta \neq 0$, the model is not stable

4.6 Model Specification

The model employed to determine the remittance and per capita growth nexus is expressed in (1):

$$\zeta = f(\text{IREM}, \text{ER}, \text{OP}, \text{INV}) \quad (1)$$

Where ζ represents per capita growth, IREM is remittances, ER is exchange rates, OP is oil price, and INV means investment proxy gross fixed capital formation.

Log linearising (1), with the exemption of ζ , the equation becomes:

$$\zeta_t = \eta_0 + \eta_1 \text{IREM} + \eta_2 \text{ER} + \eta_3 \text{OP} + \eta_4 \text{INV} + \xi_t \quad (2)$$

Where $\eta_0, \eta_1, \eta_2, \eta_3$, and η_4 are parameters and ξ_t is the error term of the model. The a priori of the coefficients in the model are such that $\eta_1, \eta_2, \eta_3, \eta_4 > 0$.

Following earlier studies (Kutu and Ngalawa, 2016; Pesaran, et al., 2001; Giles, 2013), that argued in favour of ARDL to be employed if variables are integrated of both I(1) and I(0) combined, this study is analysed using the ARDL model. Beside the fact that earlier studies suggested ARDL for the combination of I(1) and I(0) order of integrations, ARDL model is an advanced econometric technique that is appropriate for time-series data (Jebran, Abdullah et al., 2016). In addition, the ARDL model is appropriate for very small sample (Jebran et al., 2016; Pesaran et al., 2001). Thus, the sample of the study fits into the acceptable range to employ ARDL. The ARDL also includes Bounds testing carried out first to decide long relationship among the variables used. Furthermore, the ARDL models automatically ascribe appropriate lag length to a particular variable contained in a model. This is done to obtain necessary results in a specific. Following the process that it is essential to estimate a vector autoregressive (VAR) model of order p denoted as VAR (p) for the growth equation, equation 3 presents the VAR model for the variables.

$$Y_t = \vartheta + \sum_{j=1}^k \varphi_j Y_{t-j} + \varepsilon_t \quad (3)$$

Where Y_t represents vector of the variables (ζ , IREM, ER, OP, and INV), the constant term is represented with ϑ , φ_j is a matrix of VAR considerations for lag j, and ε_t is the error term. It is expected that the explained variable is integrated of I(1), while the rest variables are integrated of I(0) and I(1) combined. Consequently, the vector error correction (VEC) model is expressed as:

$$Y_t = \vartheta + Y_{t-1} + \psi \sum_{j=1}^{k-1} \psi_j \Delta Y_{t-j} + \varepsilon_t \quad (4)$$

Where Δ is the first difference and ψ is the longrun multiplier such that:

$$\begin{bmatrix} \psi_{yy} & \psi_{yx} \\ \psi_{xy} & \psi_{xx} \end{bmatrix}$$

Furthering this analysis, the study carries out the F-statistic or Wald test to decide the joint significance of the variables in the long-run. Consequently, using bound testing procedure, the null hypothesis and alternative hypothesis for the long-run relationship among variables are specified as:

$$\begin{aligned} \eta_0 = \eta_1 = \eta_2 = \eta_3 = \eta_4 &= 0 \\ \eta_0 \neq \eta_1 \neq \eta_2 \neq \eta_3 \neq \eta_4 &\neq 0 \end{aligned}$$

To compute the F-statistic, the procedure requires a comparison of estimated F-test value against the tabulated critical values (Pesaran et al., 2001). This is proposition is premised on the yardstick that the variables employed for the study

should be a combination of I(1) and I(0) as revealed in Tables 2a and 2b. However, decision formation on F-statistic is that, reject null hypothesis, if the estimated value of F-statistic is more than the upper bound. This suggests that there is stable long-run relationship in the employed variables. Inversely, the null hypothesis is accepted, if the estimated value of F-Statistic is less than the lower bound, suggesting that there is no long-run relationship among the variables. Consequently, decision formation will be based on estimation from the short-run. Having found a long-run relationship among the variables, it is essential to estimate the ARDL model and long-run coefficient between remittances and per capita growth as specified below:

$$\zeta_t = \eta_0 + \eta_1 \zeta_{t-1} + \eta_2 \text{IREM}_{t-1} + \eta_3 \text{ER}_{t-1} + \eta_4 \text{OP}_{t-1} + \eta_5 \text{INV}_{t-1} + \xi_t \quad (5)$$

Where $\eta_0, \eta_1, \eta_2, \eta_3, \eta_4$, and η_5 are parameters and ξ_t is the error of the model. The a priori of the coefficients in the model are such that $\eta_0 > 0, \eta_1 > 0, \eta_2 > 0, \eta_3 > 0, \eta_4 > 0$, and $\eta_5 > 0$.

From (5), the ECM is derived as presented in (6) which is used to obtain the short-run relationship.

$$\zeta_t = \eta_0 + \eta_i \sum_{i=1}^p \Delta \zeta_{t-1} + \eta_j \sum_{j=1}^q \Delta \text{IREM}_{t-j} + \eta_k \sum_{k=1}^q \Delta \text{ER}_{t-k} + \eta_l \sum_{l=1}^q \Delta \text{OP}_{t-l} + \eta_m \sum_{m=1}^p \Delta \text{INV}_{t-m} + \phi \text{ECM}_{t-1} + \xi_t \quad (6)$$

Where $\eta_i, \eta_j, \eta_k, \eta_l$, and η_m are short-run coefficients, Δ is the first difference of the used variables, and ϕECM_{t-1} is the error correction term. The error correction term accounts for the disequilibrium adjusted in the long-run.

5. Results

This section presents findings from the study. The study examined, using time series data, the impact of remittances on per capital growth in Nigeria. Consequently, the results are presented in this section. Table 1 presents the descriptive statistics of the time series variables used in the study. According to the table, the average per capital (ζ) oil price (OP), international remittances (IREM), investments (INV) and exchange rates (ER) are 1329.24, 43.02, 8.90, 35.74 and 106.92. This study majorly focuses on remittances and per capita growth because they constitute core variables in this study. The average value of ζ lies nearby the upper end of the distribution. Furthermore, it reveals the standard deviations of ζ , OP, IREM, INV, and ER from their respective average values yearly standing at 876.17, 29.84, 1.38, 19.18, and 98.94.

Table 1: Descriptive statistics

	EXR	INV	IREM	OP	Z
Mean	106.9272	35.74121	8.902552	43.02352	1329.246
Median	106.4643	32.04361	9.074669	29.31750	955.0451
Maximum	380.2556	89.38613	10.38580	111.9596	3098.986
Minimum	0.617708	14.16873	6.384627	12.71917	270.2240
Std. Dev.	98.94459	19.18636	1.387377	29.84654	876.1759
Skewness	0.928562	1.062168	-0.430168	1.086645	0.446371
Kurtosis	3.368702	3.822823	1.736621	3.019278	1.739752
Jarque-Bera	5.974757	8.649741	3.893843	7.872598	3.975355
Probability	0.050419	0.013235	0.142713	0.019520	0.137013
Sum	4277.088	1429.648	356.1021	1720.941	53169.82
Sum Sq. Dev.	381811.2	14356.54	75.06774	34741.83	29939684
Observations	40	40	40	40	40

Source: Authors' construct (2022), from data obtained from WDI.

Time series data comprising per capital growth, remittances, exchange rates, oil prices, and investments have been used in this study. Following the understanding that time series data are usually associated with unit root, the study carried out stationarity tests to avoid the possibility of generating spurious analysis. Using the two (the Augmented Dickey Fuller (ADF) and Phillips and Perron (PP) methods frequently used in the literature, the study first checked for stationarity of the data used (see Table 2a and 2b). As presented in Table 2a and 2b, the results reveal that the variables are stationary in I (0) and I (1) and no one in I (2). Considering the mixture of I (0) and I (1) coupled with small sample size of data accounts for the use of ARDL model.

Table 2a: Augmented Dickey Fuller (ADF)

ADF (individual intercept)				ADF (individual intercept and trend)			
Variable	Order of integration	t * statistics	P-value	Order of integration	t * statistics	P-value	
Z	I(1)	-7.037708	0.0000***	I(0)	-3.83875	0.0248**	
EXR	I(1)	-0.400592	0.0408**	I(1)	-4.575810	0.0040***	
INV	I(0)	-3.726546	0.0074**	I(0)	-3.610453	0.0412**	
IREM	I(1)	-6.44285	0.0000***	I(1)	-6.36127	0.0000***	
OP	I(0)	-3.454601	0.0162**	I(0)	-4.189538	0.0144**	

Note: ***, ** and *, respectively, represent statistical sign at 1, 5 and 10%.

Source: Authors' construct (2022), from data obtained from WDI.

Table 2b: Phillip Peron (PP)

ADF (individual intercept)				ADF (individual intercept and trend)			
Variable	Order of integration	t * statistics	P-value	Order of integration	t * statistics	P-value	
Z	I(1)	-6.640747	0.0000***	I(1)	-6.93611	0.0000***	
EXR	I(1)	-3.663743	0.0395**	I(1)	-4.534358	0.0495**	
INV	I(0)	-3.726546	0.0074**	I(0)	-3.137851	0.0481**	
IREM	I(1)	-6.44285	0.0000***	I(1)	-6.36127	0.0000***	
OP	I(0)	-3.454601	0.0162**	I(0)	-4.446863	0.0498**	

Note: ***, ** and *, respectively, represent statistical sign at 1, 5 and 10%.

Source: Authors' construct (2022), from data obtained from WDI.

Selecting a suitable lag length for the ARDL model, the study considers the recognised criteria in the literature, the Schwartz-Bayesian information criterion (SIC) and Akaike information criterion (AIC). These criteria are normally used to determine optimal lag, in a single variable, having distributed lag model (Rotimi et al., 2021; Rotimi, Ngalawa, Adebayo, 2019). The criteria for the lag selection submit that the lag with the least criterion be given consideration. The order of lag criteria includes Hannan Quinn information criterion (HQ), Akaike information criterion (AIC), Schwarz information criterion (SC) and final prediction error (FPE). For this model, optimal lag 1 is the most preferred by each criterion (see Table 3). It is specified using asterisks.

Table 3: Optimal lag length selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-795.4301	NA	4.25e+12	43.26649	43.48418	43.34324
1	-599.4349	328.4243*	4.17e+08*	34.02351*	35.32966*	34.48399*
2	-576.5213	32.20285	5.05e+08	34.13629	36.53090	34.98050
3	-549.5041	30.66819	5.59e+08	34.02725	37.51032	35.25519

Source: Authors' construct (2022), from data obtained from WDI.

The Bound test is conducted to determine whether the selected variables (per capita growth, international remittances, exchange rates, oil price and investments) exhibit relationship in the long-run. The result is presented in Table 4. It is revealed from the results that estimated F-statistic value is greater than the tabulated upper bound values, suggesting a stable long-run relationship amongst variables. Hence, null hypothesis is rejected.

Table 4: Bounds Test

Country	Variable	F-statistic value	Lag length	Significance level	Bound Critical Values	
					I(0)	I(1)
Nigeria	Per capita growth	5.3628	1	1%	4.3811	4.5312
				5%	3.9432	4.1345
				10%	2.9912	3.4239

Source: Authors' construct (2022), from data obtained from WDI.

Finding from the bound test reveals long-run relationship in the variables. So, the study proceeds to estimate the long-run relationship among the selected variables. The empirical finding showing the long-run relationship is depicted in Table 5. The findings show a positive relationship between remittances and per capita growth. This shows that an increase in the volume on international remittances lead to a favourable impact on per capita growth and vice versa. In addition, the results reveal a positive relationship between remittances and investments. This suggest that as remittances increase, investments are encouraged which in turn favourably impact output and per capita growth. This result is in line with Jebran et al., (2016) and several other recent studies (Kudaisi et al., 2021; Bettin et al., 2014; Jouini et al., 2021). In addition, oil prices reveal a positive and statistically significant relationship with per capita.

Table 5: Long-run equation (ARDL)

Variable	Coefficient	Standard Error	t-Statistic	Prob.*
EXR(-1)	1.114603	0.394047	2.828604	0.0000
INV	0.532389	0.361227	1.473501	0.0468
IREM	2.514828	2.653259	0.947826	0.0331
OP	0.756384	0.364218	2.076734	0.0001

Source: Authors' construct (2022), from data obtained from WDI.

Following the long-run relationship results, the study estimates the short-run relationship among the variables, using an error correction model (ECM) through ARDL to estimate error correction term to know the speed of adjustment as in the long-run as presented in Table 6. The finding reveals a short-run relationship between remittances and per capital growth. It indicates that remittances and per capita growth are related positively. The results also reveal that exchange rates, oil price and investment are positively related with per capita GDP in the short-run. The relationship among the variables (exchange rates, investment, oil price and remittances) with per capita growth is statistically significant. This finding implies that, as exchange rates, investment, oil price and remittances increase by one unit each, cause per capita growth to increase by 111%, 53%, 251% and 75% respectively. This is in line with theory and early studies (Jebran et al., 2016; Kudaisi et al., 2021; Bettin et al., 2014; Jouini et al., 2021). Furthermore, the results show that $\phi ECT(-1)$ is significantly negative, confirming that there is a stable long-run relationship among the selected variables. The speed of adjustment as revealed by $\phi ECT(-1)$ shows that the adjustment made over the long-run, annually is 28.83%.

Table 6: Short-run equation (ARDL)

Variable	Coefficient	Standard Error	t-Statistic	Prob.*
EXR(-1)	1.003686	0.047493	21.13356	0.0000
INV	0.389575	0.227936	1.709141	0.0468
IREM	3.528863	3.592465	0.982296	0.0331
OP	0.846567	0.187336	4.518968	0.0001
Z	0.009518	0.010153	0.937457	0.0355
$\zeta(-1)$	0.012317	0.006700	1.838323	0.0750
C	-0.583918	34.10379	-0.017122	0.0864
ECT(-1)	-0.288319	0.791232	-0.364392	0.0464

Source: Authors' construct (2022), from data obtained from WDI.

The study, assessing the strength of the model selection criteria, shows the strength of the AIC model selection compared with other models which consists of the Schwarz and HQIC criterion used in the ARDL model. It establishes the short-run relationship of ARDL model. The study employs a criteria graph to decide the topmost sixteen ARDL models. Subsequent to the prevalent model benchmark analysis, the smaller the AIC value, the better the ARDL model (Bakar et al., 2013; Giles, 2013). Hence, the ARDL criteria graph with ARDL (1, 0, 0, 0, 1) and 7.83 AIC value is the most preferred because it has the least value. Equally, the ARDL criteria graph with (1, 1, 1, 1, 0) and 8.12 AIC value is the minimum preferred because it has the maximum AIC value.

Akaike Information Criteria

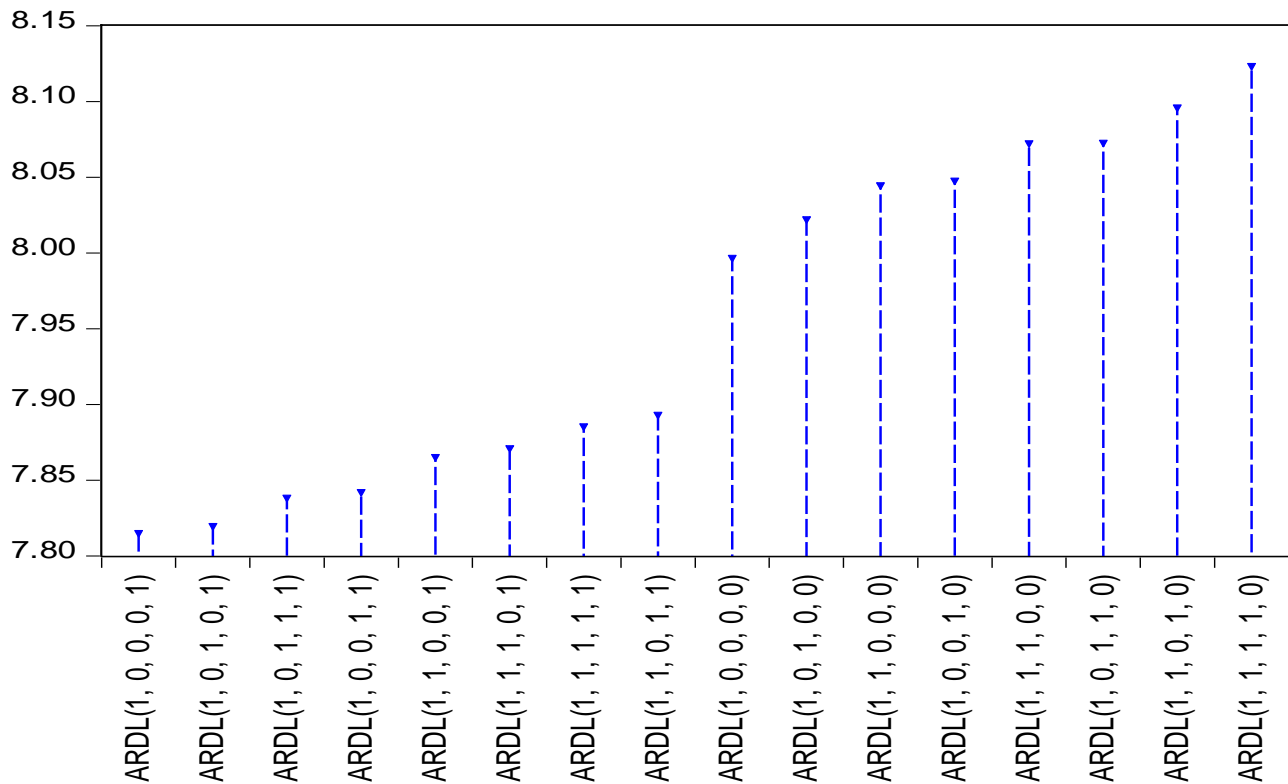


Figure 2: ARDL Model Selection Strength

Source: Authors' construct (2022), from data obtained from WDI.

The serial correlation test is carried out to investigate problem of serial correlation in the model. As depicted in Table 7, there is no evidence of serial correlation since the probability of the F-statistics is 0.1, greater than the 0.5 benchmark. Hence, the null hypothesis should be accepted. In another word, it means that the parameter estimates in the model have no autocorrelation problem.

Table 7: Breusch-Godfrey Serial Correlation LM Test

F-statistic	0.036359	Prob. F (1,32)	0.8500
Obs*R-squared	0.045397	Prob. Chi-Square (1)	0.8313

Source: Authors' construct (2022), from data obtained from WDI.

Table 8 presents the heteroskedasticity results. It shows that the F-statistics and chi square are insignificant at 5%. This means that there is no heteroskedasticity in the model. It could therefore be concluded that the model is reliable and valid.

Table 8: Heteroskedasticity Result

F-statistic	10.67566	Prob. F (6,33)	0.4201
Obs*R-squared	26.39931	Prob. Chi-Square (6)	0.2102
Scaled explained SS	30.14120	Prob. Chi-Square (6)	0.0000

Source: Authors' construct (2022), from data obtained from WDI.

The normality result presented in Figure 4.31 indicates normality in the distribution. This is evident in the Jarque-Bera statistics revealing a statistically insignificant value, 5 per cent (that is, 5.765841). This is buttressed with probability value estimated as 0.015. Residuals are normally distributed suggesting that the data sets are modelled appropriately.

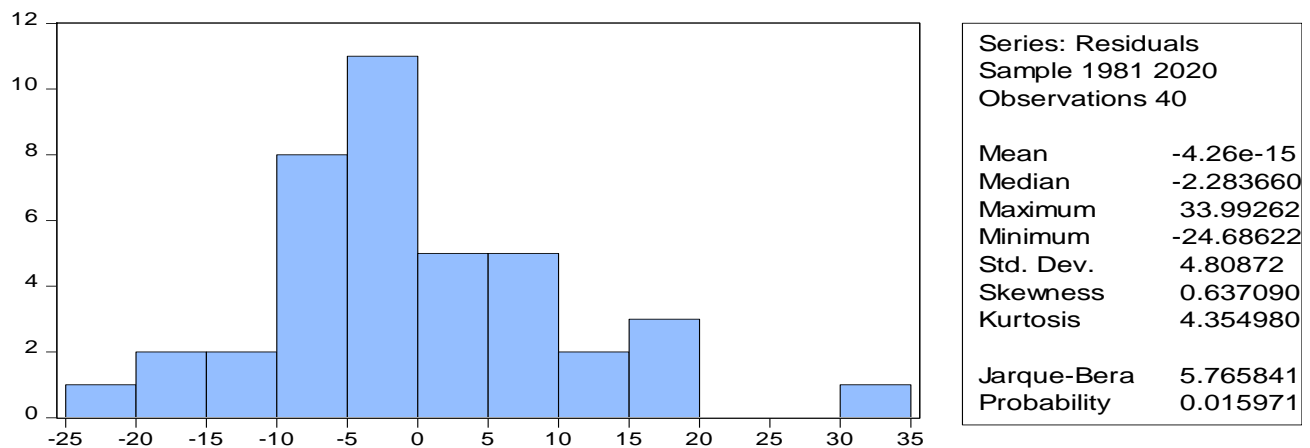


Figure 3: Result of Normality Test

Source: Authors' construct (2022), from data obtained from WDI.

Figure 4 presents the result of the CUSUM stability test for the model. It reveals stability in the model because the line that captures our data falls between the 5 per cent significant level.

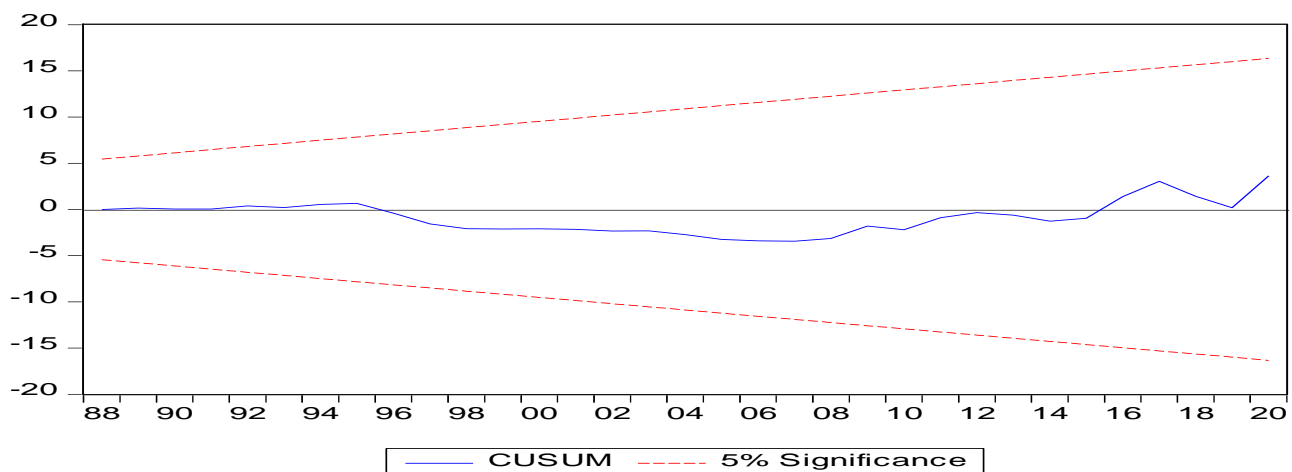


Figure 4: CUSUM Test

Source: Authors' construct (2022), from data obtained from WDI.

6. Conclusions and Recommendations

This study unpacks the international remittances and per capital growth nexus in Nigeria from 1980 to 2020. Specifically, the study seeks to make provisions for further underpinnings on the relationship between remittances and per capital growth both in the short-run and long-run. The study carried out the ARDL bounds testing model to ascertain the short-run and long-run relationship between per capital growth and other variables (exchange rates, investment, oil price and remittances). Findings reveal statistically significant positive short-run and long-run relationship among the variables. Specifically, various results support positively strong remittances and per capita growth nexus in Nigeria. This further suggests that higher international remittances enhance per capita growth both in the short-run and long-run in Nigeria. Various diagnostic tests support the reliability and appropriateness of the model employed. This suggests that remittances are sources of external financing and eventually, it is a means to economic growth and also may also help to fill fiscal deficit gap. In addition, the normality test results indicate that the model is normal. In view of these findings and the relevance of remittances as established in this study, the study recommends that government should, through sound policy option, encourage remittances influx. This could be realised by creating viable relationship among international communities that largely account for remittance inflow into Nigeria. Prudent and optimal management of remittances inflow through the appropriate monetary authority is recommended. This may include formulating policy that will ease remittance inflow and remove unnecessary barriers to inflow of remittances. Without mincing words, profitable

investment of remittances is strongly recommended. Further study is suggested to study the impact of remittances on consumption.

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Modelling Stock Market Prices Using the Open, High and Closes Prices. Evidence from International Financial Markets

Samuel Tabot Enow[†]

Research Associate, The IIE Vega School, South Africa, Email: enowtabot@gmail.com

ARTICLE INFO	ABSTRACT
<p>Article History</p> <p>Received 13 January 2023; Accepted 08 February 2023</p> <p>JEL Classifications G1, G2, G4</p> <p>Keywords: Modelling stock price; Bayesian Model; Stock market; Financial markets; VAR</p>	<p>Purpose: Modelling security prices seem to be an ending debate in finance literature due to no clear consensus on behavioral patterns. Knowledge of stock price movement has always been an important source of information that is much needed in asset pricing and trading strategies. The aim of this study was to model stock market prices using six international markets as a sample.</p> <p>Design/methodology/approach: This study made use of the Bayesian Time-Varying coefficient for a five-year period from January 2, 2018, to January 2, 2023.</p> <p>Finding: The findings of this study revealed that there is strong empirical evidence that the returns of a security can be modelled using the open, high and low prices.</p> <p>Research limitations/implications: This implies that the drift in stock price movement can be better explained by observing the lag values of the open, high and low prices which may be an important tool for short term traders and incorporated in volatility estimation. Also, the lag values of the open, high and low price movements explain more than 98% of changes in the closing price.</p> <p>Originality/value: As per the author's knowledge, this study is the first to model stock market prices using the open, high and low prices for multiple international markets.</p>

1. Introduction

At the beginning of every year, analysts and academics often make gloomy forecasts about the expected performance of stock markets and economic conditions. These forecasts often cause panic and are mostly incorrect. To this end, there is no concrete approach in predicting stock market prices due to the consistent poor forecasts in stock market prices. Swedroe (2018) compiled a list of predictions made by analysts and academics for over a 7-year period, he diligently tracked these predictions and reported on their results. These forecasts were mainly 69 sure predictions from 2010 to 2018 and only 32% materialized as expected (Swedroe, 2018). The forecasted values of security prices were also studied in-depth in a paper by Bailey et al. (2018) where the authors examined 6627 forecasts made by 68 analysts. The findings revealed that 48% of those forecasts were correct and 66% had an accuracy score of less than 50% (Bailey, et al., 2018). However, there are some quantitative measures that have been very useful in forecasting future returns such as the Shiller cyclically adjusted price earnings ratio. According to this matrix, higher stock prices tend to be followed by lower stock returns. Also, prior literature (Mettle et al., 2014; Pacifico, 2021; Dar et al., 2022) still contends that stock prices follow a Markov process which is consistent with the weak form efficiency. Implying that to some extent, stock prices still encapsulate previous price history although the main driver is relevant new information (Liyanagamage and Madusanka, 2021). Many studies on modelling stock price have actively argued that the expected stock price changes in an infinitesimal time dt is constant and independent of past price movement. In essence;

$$\text{Expected price change} = E \left\{ \frac{1}{dt} \left[\frac{ds}{s_0} \right] \right\} = \mu \quad (1)$$

[†]Corresponding Author: Samuel Tabot Enow
Email: enowtabot@gmail.com

$$\frac{ds}{s_0} = \mu dt + \text{randomness with zero mean} \quad (2)$$

Considering the above randomness in mean return, the expected variability in stock price changes over a period should be given by;

$$VAR \left[\frac{ds}{s_0} \right] = \sigma^2 dt$$

(3)

Where VAR is the value at risk and σ^2 is the variance. It is therefore important to note that forecasting stock market prices can drift higher or lower than the expected value making it difficult to successfully categorize the price behaviour. Consequently, there is a large distribution of outcomes that are still not accounted for when making stock price forecasts. These distributions may be well explained when the open, high and low prices are included in the forecast of stock prices. Hence this study seeks to answer the following research question; Should the open, high and low prices be used to model stock market prices? The main aim to this study is to empirically ascertain whether the open, high and low prices can be used as good predictors of closing prices hence stock market returns. In so doing, this study makes a significant contribution on modelling stock prices and to a broader extent, modelling volatility of stock prices.

2. Literature review

Stock price modelling can be extremely difficult due to the widely accepted concept of market efficiency and weak form efficiency. In essence, stock prices are assumed to follow a Markov process and move only with new information (Enow, 2022). However, the development of stochastic processes has proven otherwise. Empirical research reveals that the distribution of stock price movements can be modelled to some extent. There is a rich literature on modelling stock prices, but almost if not all the studies used closing prices to forecast price movement. Table 1 highlights the most recent studies.

Table 1: Review of prior studies

Study (Author & year of study)	Model	Period	Variables	Findings
Ugurlu et al. (2014)	GARCH	January 8, 2001, to July 20, 2002	Logarithm of closing price relative ($\frac{p_x}{p_{x-1}}$)	GARCH model is a reliable predictor of closing prices.
Boateng et al. (2015)	ARCH/GARCH model	Not disclosed	Closing price relative ($\frac{p_x}{p_{x-1}}$)	Constant variance in closing price returns.
AL-Najjar (2016)	ARCH, GARCH, and EGARCH	Jan. 1, 2005 - Dec.31 2014.	Closing price relative ($\frac{p_x}{p_{x-1}}$)	The author forecasted persistence in volatility due to asymmetry effect.
Adewuyi (2016)	Exponential Weighted Moving Average	June 13, 2006 – December 1, 2014.	Logarithm of closing price relative ($\frac{p_x}{p_{x-1}}$)	High probability of decreasing stock prices from 2015
Kaya & Güloğlu (2017)	FIAPARCH & GARCH	January 1, 2002 – April 29, 2016	The logarithm difference in previous closing prices ($\ln P_{i,t} - \ln P_{i,t-1}$)	The FIAPARCH model is a good predictor of volatility than the GARCH model.
Kuhe (2018)	GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1)	July 3, 1999 – June 12, 2017	Logarithm of closing price relative ($\frac{p_x}{p_{x-1}}$)	The EGARCH (1,1) model was a better predictor of market volatility than the other GARCH models.
Yatigammana et al. (2018)	Autoregressive Moving Average	January 16, 2014 - April 15, 2014	The logarithm difference in previous closing prices	Only 78 and 91 percent of the stock price can be

			$(\ln P_{i,t} - \ln P_{i,t-1})$	estimated
Ghani & Rahim (2019)	ARMA (1,0) - GARCH	January 4, 2010 – December 29, 2017.	Daily closing prices	The ARMA (1,0) – GARCH model is the best predictor of market volatility.
Schmidt (2021)	GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1)	February 19, 2020 – April 7, 2021.	The logarithm difference in previous closing prices $(\ln P_{i,t} - \ln P_{i,t-1})$	GARCH (1.1) model should not be used to forecast volatility as it had the worst performance.
Liyanagamage & Madusanka (2021)	Auto Regressive Moving Average	2009 - 2019	Past stock prices	Past stock prices can be reliably used to predict future prices

Source: Author's construct

Table 1 above provides some interesting findings. It can be observed that GARCH models and closing security prices are predominantly used to analyse and forecast market volatility. Although these studies may be relevant, daily open, high and low stock prices have not been widely used in any of these analysis. Therefore, this study is aimed at advancing the frontier of stock price behaviour forecasting by examining the effect of lag the values for the open, high and low market prices on the closing prices in international stock markets.

3. Data & methodology

In the past decade, attention has been given to many Value at Risk (VAR) models which is suitable for allowing coefficients to change over time. One of such models is the switching VAR which enables discrete occasional changes to coefficients. An alternative model to switching VAR is the Time Varying coefficient VAR (TVC VAR) which allows continuous smooth changes to coefficient with continuum of variables (Amadi et al. 2022). Whilst the large coefficients space of TVC VAR has attractive properties from a modelling perspective, it can also lead to difficulty in estimations. To this end, this study used a Bayesian Time-Varying coefficient (BTVC). A BTVC VAR has become the de factor approach to estimating time varying coefficients due to its superior forecasting technique in placing more weight in the lag values of one or more variables (Karlsson and Österholm, 2020). This model allows credible heterogeneity parameters that are suitable for modelling. In essence, the model integrates latent variables together with their probability distributions which enhances modelling inferences. As an additional benefit, the BTVC model incorporates both unconditional distribution and latent moments of the independent variables. As such, it is very useful in exploring relationships between multiple variables, hence was deemed appropriate for this study.

In its simplest form, A BTVC model is given by;

$$\ln(Close) = l_t + s_t + \sum_{t=1}^n \ln(open_{t,t-1}) \beta_{t,t-1} \sum_{t=1}^n \ln(high_{t,t-1}) \beta_{t,t-1} \sum_{t=1}^n \ln(low_{t,t-1}) \beta_{t,t-1} + \varepsilon \quad (4)$$

Guhaniyogi et al. (2022)

Where l_t is the trend, s_t is the seasonality treated as a regression on Fourier series and $\beta_{t,t-1}$ is the time varying coefficient. The open, high, low and closing prices for six international financial markets namely, Johannesburg Stock Exchange (JSE), Nasdaq Index, the French Stock Market Index (CAC 40), the Nikkei Stock Average (Nikkei 225), the German blue-chip companies (DAX) and the Borsa Istanbul Index 100 (BIST) were sourced from Yahoo finance. The sample period was from January 2, 2018, to January 2, 2023.

4. Results and discussion

The results of the analysed data from the sampled financial markets are presented below.

Table 2: model level descriptions

		HIGH	LOW	OPEN	P-Value (F-stats)
JSE	Adjusted R-square	0.995076	0.995034	0.993669	
	F-Stats	31499.49	31231.65	24467.97	0.000*
Nasdaq	Adjusted R-square	0.980426	0.977317	0.980902	
	F-Stats	1547.438	1331.289	1586.814	0.000*
CAC 40	Adjusted R-square	0.993923	0.992438	0.995764	
	F-Stats	26106.58	20949.29	37524.85	0.000*
Nikkei 225	Adjusted R-square	0.863465	0.863975	0.908160	
	F-Stats	190.7239	191.5479	297.6551	0.000*

DAX	Adjusted R-square	0.992476	0.990618	0.994502	
	F-Stats	20825.61	16670.50	28556.17	0.000*
BIST	Adjusted R-square	0.995326	0.995086	0.995218	
	F-Stats	32261.35	30678.13	31534.02	0.000*

Source: Author's construct

From table 2, the variability of the closing prices can be well explained by movements in the open, high and low prices. This is evident in the adjusted R square values that are close to one in all the stock markets under consideration. More specifically, all the adjusted R-square values are more than 98% indicating high levels of explanatory power. It can be suggested that the opening, high and low prices provides a meaningful explanation for the variability of the closing prices and adding additional variable may not add any value. Based on these findings, closing prices in financial markets have a high correlation with the opening, high and low prices. The F-stat test results strengthen further the explanatory effect of the opening, high and low prices on the closing prices. The p-values of the F-stats for all the financial markets under consideration are significant at 5% indicating a perfect fit of the model.

Tables 3, 4, 5,6,7 and 8 in the appendix provide the output results of the BTVC VAR estimates. From these results, the lag values of the Bayesian coefficient have positive and negative signs indicating a two-way impact. Hence, the lag values of the opening, high and low prices affect the closing prices positively and negatively. Most importantly, the 2-day lag values of the open, high and low prices are significant in all the sampled financial markets with the exception of the Nasdaq in table 4 which may signal some form of market efficiency (Enow, 2021). This means that proper analysis of the open, high and low prices for the past 2 days can be used as a guide to forecast the closing prices. In essence, todays candlestick charts of the high and low prices may provide significant information on the price movement for the next 2 days. However, the 1-day lag values of the open, high and close are insignificant with the exception of the JSE in table 3. By implication, prior information on the open, high and low price movements cannot be used as a good guide to predict the variability of the closing price distribution for the next day. These findings are supported by the regression results in tables 9, 10, 11, 12, 13 and 14 in the appendix which also revealed significant adjusted R-square values as high as 99%. The regression estimates in tables 9 to 13 portrays a significant positive relationship between the high/low prices and the closing prices. However, an inverse relationship was observed between the high/low prices and closing prices in the BIST as shown in table 14. This implies that the high and low prices move in the same direction with the closing prices in the JSE, Nasdaq Index, CAC 40, Nikkei 225 and DAX but vice versa in the BIST. In so doing, observing the price distribution of the open, high and low prices can provide a vivid understanding of the closing price returns.

5. Conclusion

Prior literature suggest that modelling stock prices is often based on observing historical returns and the concept of efficient market hypothesis where prices are assumed to follow a random pattern. The purpose of this study was to model closing prices using the open, high and low prices for a 5-year period using the BTVC VAR model. The findings of this study revealed that the closing price return in financial markets can also be modelled using the open, high and low prices. In essence, observing the market price movement for the previous 2 days' period provides a good indication of the closing market price. The shortcomings of conventional price modelling methods may be overcome by including the open, high and low price movements which may provide a more robust approach. This is in sharp contrast to a relatively outdated study by Floros (2009) who found that high and low prices overestimate future market return due to clustering effect. In this study, the lag values of these open, high and low price movements rather explained more than 98% of the changes in closing price returns. From these findings, incorporating drift in stock price movement can be better explained by observing the open, high and low prices which may be an important tool for short term traders and market speculators to predict the possible direction of the market. This study advances the frontier in forecasting stock price movements by using different variables and methods in modelling returns compared to other studies in prior literature (Ugurlu et al., 2014; Boateng et al., 2015; AL-Najjar, 2016; Adewuyi, 2016; Kaya and Güloğlu, 2017; Kuhe, 2018).

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Appendix

Table 3: JSE Bayesian VAR Hyper-parameters

	CLOSE	HIGH	LOW	OPEN
CLOSE (-1)	0.890350 (0.04178)*	0.548464 (0.04113)*	0.573229 (0.04156)*	0.609983 (0.05612)
CLOSE (-2)	0.042413 (0.03606)*	-0.01347 (0.03537)*	-0.00969 (0.03574)*	-0.03489 (0.04827)*
HIGH (-1)	0.000764 (0.04380)*	0.460691 (0.04321)*	0.010276 (0.04359)*	0.101939 (0.05889)
HIGH (-2)	0.016656 (0.03241)*	0.036159 (0.03206)*	-0.05266 (0.03227)*	0.003122 (0.04361)*
LOW (-1)	0.024428 (0.04083)*	0.057226 (0.04021)*	0.461215 (0.04072)*	0.091468 (0.05492)
LOW (-2)	0.003379 (0.03125)*	-0.01929 (0.03079)*	0.060412 (0.03124)*	0.033337 (0.04206)*
OPEN (-1)	0.002940 (0.03099)*	-0.07176 (0.03055)*	-0.09114 (0.03087)*	0.167827 (0.04177)*
OPEN (-2)	0.015675	0.006452	0.038647	0.018689

Source: Author's construct

Table 4: Nasdaq Bayesian VAR Hyper-parameters:

	CLOSE	HIGH	LOW	OPEN
CLOSE (-1)	0.959637	0.239380	0.261953	0.454729

	(0.07228)	(0.05809)	(0.06085)	(0.07239)
CLOSE (-2)	-0.00447	-0.02781	-0.02287	-0.04633
	(0.04610)*	(0.03690)*	(0.03865)*	(0.04603)*
HIGH (-1)	0.031383	0.895129	0.112579	0.035407
	(0.09338)	(0.07574)	(0.07890)	(0.09399)
HIGH (-2)	0.041898	-0.0078	-0.00932	-0.03322
	(0.05298)	(0.04308)*	(0.04480)*	(0.05336)
LOW (-1)	0.002626	0.113122	0.878964	0.055935
	(0.08937)	(0.07210)	(0.07594)	(0.08995)
LOW (-2)	-0.02268	-0.04592	-0.03908	-0.05634
	(0.05145)	(0.04154)*	(0.04384)*	(0.05183)
OPEN (-1)	-0.03308	-0.15945	-0.17793	0.603348
	(0.07846)	(0.06336)	(0.06636)	(0.07948)
OPEN (-2)	-0.00288	-0.02192	-0.02602	-0.02486
	(0.04231)*	(0.03417)*	(0.03579)*	(0.04292)*

Source: Author's construct

Table 5: CAC 40 Bayesian VAR Hyper-parameters

	CLOSE	HIGH	LOW	OPEN
CLOSE (-1)	1.045883	0.556513	0.610716	0.800806
	(0.05392)	(0.04773)*	(0.05278)	(0.05333)
CLOSE (-2)	0.052203	0.045074	0.020143	-0.01447
	(0.03990)*	(0.03518)*	(0.03890)*	(0.03933)*
HIGH (-1)	0.037950	0.658006	0.007562	-0.01025
	(0.06023)	(0.05355)	(0.05901)	(0.05973)
HIGH (-2)	-0.04094	0.027967	-0.08021	-0.01189
	(0.04097)*	(0.03653)*	(0.04017)*	(0.04066)*
LOW (-1)	-0.00241	0.003993	0.662486	0.013161
	(0.05628)	(0.04988)*	(0.05531)	(0.05580)
LOW (-2)	0.039932	0.001897	0.059591	0.003379
	(0.03808)*	(0.03377)*	(0.03754)*	(0.03778)*
OPEN (-1)	-0.13679	-0.29643	-0.29378	0.215390
	(0.05910)	(0.05244)	(0.05797)	(0.05880)
OPEN (-2)	-0.00024	1.48E-07	0.009406	0.000952
	(0.03528)*	(0.03133)*	(0.03462)*	(0.03517)*

Source: Author's construct

Table 6: Nikkei 225 Bayesian VAR Hyper-parameters

	CLOSE	HIGH	LOW	OPEN
CLOSE (-1)	0.965978	0.140210	0.132472	0.293061
	(0.07817)	(0.06885)	(0.07229)	(0.06630)
CLOSE (-2)	-0.00395	-0.015	-0.01644	-0.04405
	(0.04574)*	(0.04018)*	(0.04218)*	(0.03872)*
HIGH (-1)	-0.04395	0.877895	0.024000	0.004473
	(0.09163)	(0.08162)	(0.08515)	(0.07818)
HIGH (-2)	-0.00525	-0.0144	-0.02221	-0.04119

	(0.05013)	(0.04474)*	(0.04661)*	(0.04279)*
LOW (-1)	-0.01385	0.073738	0.930166	0.096103
	(0.08513)	(0.07535)	(0.07958)	(0.07261)
LOW (-2)	-0.00555	-0.01949	-0.02523	-0.05181
	(0.04773)*	(0.04227)*	(0.04472)*	(0.04074)*
OPEN (-1)	0.047400	-0.09232	-0.05997	0.734024
	(0.09169)	(0.08124)	(0.08527)	(0.07871)
OPEN (-2)	-0.0317	-0.02978	-0.04092	-0.04729
	(0.04914)*	(0.04353)*	(0.04570)*	(0.04223)*

Source: Author's construct

Table 7: DAX Bayesian VAR Hyper-parameters

	CLOSE	HIGH	LOW	OPEN
CLOSE (-1)	1.050835	0.539426	0.580424	0.793156
	(0.05503)	(0.04873)*	(0.05335)	(0.05538)
CLOSE (-2)	0.060219	0.037440	0.022927	-0.0235
	(0.03999)*	(0.03527)*	(0.03861)*	(0.04011)*
HIGH (-1)	-0.02863	0.637312	0.014143	-0.01338
	(0.06232)	(0.05543)	(0.06048)	(0.06289)
HIGH (-2)	-0.00566	0.040917	-0.06607	-0.00643
	(0.04128)*	(0.03682)*	(0.04009)*	(0.04169)*
LOW (-1)	-0.00763	0.063553	0.656970	0.045978
	(0.05743)	(0.05091)	(0.05591)	(0.05794)
LOW (-2)	0.008147	-0.01633	0.031965	-0.00187
	(0.03849)*	(0.03414)*	(0.03758)*	(0.03885)*
OPEN (-1)	-0.08717	-0.31023	-0.26146	0.199959
	(0.05852)	(0.05194)	(0.05686)	(0.05925)
OPEN (-2)	0.003782	0.002168	0.016893	0.002585
	(0.03506)*	(0.03114)*	(0.03408)*	(0.03556)*

Source: Author's construct

Table 8: BIST Bayesian VAR Hyper-parameters

	CLOSE	HIGH	LOW	OPEN
CLOSE (-1)	0.973359	0.142025	0.146705	0.272355
	(0.07734)	(0.07606)	(0.07665)	(0.07722)
CLOSE (-2)	0.001777	-0.00125	-0.004	-0.00758
	(0.04264)*	(0.04188)*	(0.04219)*	(0.04253)*
HIGH (-1)	0.031811	0.948565	0.035665	-0.01041
	(0.08091)	(0.08046)	(0.08056)	(0.08129)
HIGH (-2)	0.001902	0.001383	0.006574	0.003374
	(0.04292)*	(0.04272)*	(0.04275)*	(0.04312)*
LOW (-1)	-0.01111	0.047957	0.927122	0.037829
	(0.07863)	(0.07768)	(0.07872)	(0.07894)
LOW (-2)	-0.00654	-0.01183	-0.01549	-0.01658
	(0.04264)*	(0.04215)*	(0.04276)*	(0.04282)*
OPEN (-1)	0.009129	-0.1237	-0.10211	0.720079
	(0.07689)	(0.07609)	(0.07661)	(0.07768)

OPEN (-2)	-0.00344 (0.04175)*	-0.00479 (0.04129)*	0.002034 (0.04159)*	-0.00089 (0.04221)*
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Source: Author's construct

Table 9: Nasdaq Regression estimates

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HIGH	0.960685	0.051355	18.70684	0.0000*
LOW	0.89575	0.049476	18.10481	0.0000*
OPEN	-0.85458	0.049463	-17.2769	0.0000*

Source: Author's construct

Table 10: JSE Regression estimates

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HIGH	0.713558	0.0185	38.57045	0.00000*
LOW	0.538173	0.019531	27.55435	0.00000*
OPEN	-0.25663	0.017664	-14.5287	0.00000*

Source: Author's construct

Table 11: CAC 40 Regression estimates

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HIGH	0.767979	0.019323	39.74476	0.000*
LOW	0.785461	0.017331	45.32038	0.000*
OPEN	-0.55431	0.022419	-24.7253	0.000*

Source: Author's construct

Table 12: Nikkei 225 Regression estimates

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HIGH	0.882873	0.05072	17.40691	0.000*
LOW	0.728745	0.039944	18.24436	0.000*
OPEN	-0.60777	0.043342	-14.0228	0.000*

Source: Author's construct

Table 13: DAX Regression estimates

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HIGH	0.794966	0.01961	40.53842	0.000*
LOW	0.788672	0.017172	45.92862	0.000*
OPEN	-0.58331	0.021591	-27.0161	0.000*

Source: Author's construct

Table 14: BIST Regression estimates

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HIGH	0.00	0.00	19.18211	0.000*
LOW	0.00	0.00	33.23726	0.000*
OPEN	0.00	0.00	-20.1262	0.000*

Source: Author's construct

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Bank Capital Buffers and Bank Risks: Evidence from the Namibian Banking Sector

Johannes P. S. Sheefeni¹

¹ Department of Economics, University of the Western Cape, South Africa, <https://orcid.org/0000-0001-7260-7811>

ARTICLE INFO.	ABSTRACT
Article History	Purpose: This paper analysed the effects of bank's risk on capital buffer in Namibia, in the absence of the consensus on the cyclical behavior of capital buffers.
Received 29 March 2022; Accepted 8 December 2022	Design/methodology/approach: The study employed the autoregressive distributed lag (ARDL) modelling technique on quarterly data for the period 2001 to 2019.
<i>JEL Classifications</i> G21, E32	Findings: The study found the following: First, there is a long run relationship between the dependent variable and the independent variables. Second, the study showed that the ratio of NPLs to gross total loans negatively affect capital buffers in the short run, while it positively affects capital buffers in the long run. Furthermore, return on assets and liquidity negatively affects capital buffers in both the short and long run. On the contrary, bank size in form of log of total loans positively affects capital buffers in both the short and long run.
	Research limitations/implications: The unavailability of data of a long-term span is not desirable. Moreover, the limited data of certain variables narrowed the choice of a variety of variables that could be included in the study.
Keywords: Bank, Capital Buffers, Bank Risk, Namibia, ARDL, Procyclical, Countercyclical	Originality/value: The paper contributes to the hypothesized theory of countercyclical. The policy implication from these findings is that the presence of countercyclical relationship is in support of the transition from Basel II to Basel III to mitigate the procyclical as experienced under Basel II accords as documented in the literature. Future studies should focus on using a variety of variables to assess this relationship and see whether or not the outcome will be different.

1. Introduction

The banking sector is known for the important role it plays in the economy. One such role is that of intermediation, by collecting funds from the depositors or savers and giving the funds to the borrowers in the form of loans/credit (Sadalia et al., 2017). For example, commercial banks in Namibia are relatively large and they contribute around 70% to GDP (Paavo, 2018). The banking sector in Namibia is relatively well developed and sophisticated in comparison to her peers. The Bank of Namibia (BoN), the Central Bank, regulates the banking sector and publishes supervisory reports on regular basis to inform the public on the status quo. For the most part, capital and liquidity levels remained well within the regulatory standards as prescribed. Of greater importance was the compulsory and applicable new capital rules introduced by the Central Bank (BoN) in September 2018, in accordance with Basel III. This was applicable to all banking institutions and bank-controlled companies deemed to be of national systemic importance. Therefore, this makes the subject important to study.

However, for the mere fact that banks are also business entities like any others. Their main aim is to make profits from the spread between lending and borrowing, which comes with risks. Thus, the perceived risky activities have encouraged and seen increase in government's control over the banking sector over the years (Noreen et al., 2016).

From an international perspective, this has resulted in the establishment of supervisory body for banks. According to Noreen et al., (2016), the international committee (Basel committee) was established to assume the role of supervising banking institutions. Specifically, on matters including the monitoring of the general strength of the banks as well as their risk-management skills. In executing the supervisory role, the committee is guided by the Basel

¹ Corresponding Author: Johannes P. S. Sheefeni
Email: peyavali@gmail.com

accords (I¹, II² and III³) which provided the specifications in terms of required capital adequacy for banks. Hence, the rules on bank capital forms the major part of such regulation (Tabak et al., 2011).

The after effects of the economic crises in 2008 raised concerns on the strength of capital regulations as well as the minimum capital requirements as prescribed by Basel II (2004) (Rahman et al., 2018). This is due to the fact that most countries increased their reliance on capital buffers during business cycle fluctuations. Basically, there was countercyclical behavior by the banks over the business cycle such that bank capital buffer decreases with economic upswing and increases with economic downswing. This implied that, banks that are capitalized below-par resorts to increase their capital buffer by reducing lending activities to the market rather than the usual costly way of issuing new equity. This pose a major threat to lending activities which can be considered as a bank risk and also economic stability (Kontbay-Busun & Kasman, 2015). On the other hand, literature reveals that the procyclicality⁴ nature of the financial system can also result in macroeconomic disturbances that leads to financial instability (Whyte, 2013).

There is no consensus in literature on the cyclicity of capital buffers; that is, whether bank capital buffers are generally procyclical or countercyclical in nature. However, Tabak, et al., (2011) study provides empirical evidence of countercyclical behaviour with respect to business cycle in Brazil during the period 2000-2010. On the other hand, Whyte's (2013) study on the Jamaican banking sector provides empirical evidence of procyclicality behavior with respect to business cycle in the Jamaican banking sector during the period 2000-2012. On the subject of capital buffers visa-a-visa banks' risk, Jokipii and Milne (2009) use the non-performing loan ratio to total loans and credits (NPL) to proxy the bank risk. The paper showed a negative relationship between capital buffers and banks' risk. On the contrary, Kontbay-Busun & Kasman (2015) results suggest a positive effects of banks' risk on capital buffer. This shows that there are mixed findings in this regard, too.

In the absence of a consensus on the cyclical behavior of capital buffers, this study specifically focuses on the relationship between capital buffers and banks' risk. Particularly, the study analyses the effect of banks' risk on capital buffers. The paper is comprised of the following sections in the order of literature review, methodology, empirical analysis, results and the conclusion.

2. Review of Literature

2.1 Theoretical Review

There is literature that links the bank capital buffers, bank risk and economic cycle. This subsection reviews the theories and some of the empirical studies on the subject matter. In this study, two theories were reviewed, namely, the moral hazard theory and the charter value theory.

Moral hazard refers to a situation in which one party decides to take risk knowing that someone else will bear the cost if things go wrong (Hossain & Chowdhury, 2015). The moral hazard theory presents two possibilities. First, the theory hypothesizes that bank capital buffers, lessens the agency costs that may arise due to conflict of interest between shareholders and managers. For example, a well-capitalised bank has less incentives toward moral hazards because it tends to practice good managements traits (Danarsari et al., 2018). However, there is a contrary view that bank capital can be counterproductive. An increase in bank capital to meet the capital requirements encourages adjustments of bank's asset risk by bank managers, such that some of these acts are excessive risk-taking activities (Berger & Bouwman, 2013). That is why, a negative relationship between bank capital and bank risk refers to the 'moral hazard hypothesis' whereby banks tend to act in such a manner (Bouheni & Rachdi, 2015). Therefore, because of the abovementioned viewpoint, the enactment of capital regulations for a good purpose may also result in unanticipated unfavourable effects (Danarsari et al., 2018).

In most cases, well-capitalised banks usually hold more bank capital than it is required in order to shield themselves during downturns periods as well as to handle the default risk. This, alongside with the behaviour of managing risks, is not explained by the moral hazard theory but by the charter theory. According to Danarsari et al. (2018), the charter theory hypothesizes that banks have a potential to lose out a lot. This is due to the fact that bankruptcy results in loss of future earnings to a number of parties, including the stakeholders. For this reason, banks usually hold bank capital in excess of what is required by the regulation. This hypothesis is known as charter value - "a value placed on future assets of a business" (Danarsari et al., 2018).

In addition, the charter theory further hypothesizes that the relationship between capital buffer and risk, in the long term, can either be positive or negative. On the other hand, the same relationship in the short term depends on the degree of bank capitalization. For instance, well-capitalised banks would depict a positive relationship, while poorly-capitalised banks or approaching the required level would depict the opposite. Therefore, this theory argues that increasing the regulatory capital requirements in the short run leads to a decrease in capital buffer, which result in the same impact as direct reduction in the capital buffer (Jokipii & Milner, 2009).

2.2 Previous studies

¹ Set out the regulatory standards on market risk and credit risk.

² Further considered the operational risk and not liquidity risk.

³ New capital reforms in Basel III and negative capital buffer requirement restrictions, within a range of 0-2.5% imposed on banks.

⁴ Pro-cyclicality of the financial system can be defined as amplification of swings in the economic cycle caused by financial sector activities.

There are a number of empirical studies that support the aforementioned hypotheses as presented below. The first set of empirical studies looks at the relationship between capital buffers and bank risk. The second discussion of empirical studies looks at the relationship between capital buffers and business cycles. The third and last set of empirical studies discusses the relationship between bank risk and business cycles.

The relationship between bank risk and capital buffers has been explored in a number of studies around the world. For instance, Guidara et al. (2013) found no strong relationship between capital buffers and risk among the six largest Canadian chartered banks for the period 1982 to 2010. Therefore, they attributed excess capital held by banks to market discipline. However, Kontbay-Busun & Kasman's (2015) results revealed a positive effects of bank risk on capital buffer, but a negative effect of capital buffers on bank's default risk. These findings were for Turkey during the period 2002 to 2012. The positive relationship found in Kontbay-Busun & Kasman (2015) results are also supported by Belem and Gartner (2016) who examined 121 Brazilian banks during the period 2001 to 2011. Their study found a positive relationship between bank risk and capital buffers. This implies that the two variables moved in the same direction such that increasing capital buffers happened when there was a greater risk. Similarly, the negative relationship in the findings by Kontbay-Busun and Kasman (2015) are supported by Bouheni and Rachdi (2015) whose results revealed an inverse relationship between capital buffers and bank risk-taking. Particularly, the study showed that an increase in capital buffer was proceeded by less incentives in bank risk-taking during the period 2000 to 2013 in Tunisia. Other studies that found a negative relationship between bank risk and capital buffers were conducted on Pakistan (Noreen et al., 2016), Bagladesh (Rahman et al., 2018), and Indonesia (Danarsari et al., 2018). The negative relationship between the two variables simply implies that an increase in capital buffers leads to a reduction in bank risk-taking behavior.

Numerous studies also examined capital buffers vis-à-vis and business cycle. Guidara et al. (2013) used quarterly data for the period 1982 to 2010. The results revealed that the capital buffers for the six largest Canadian chartered banks exhibit a positive co-movement with business cycles. Similarly, Noreen et al. (2016) also examined the relationship between capital buffers and business cycle for 24 commercial banks during the period 2007 to 2012 in Pakistan. The results also revealed a procyclical behavior or a positive relationship between capital buffers and business cycle. Lastly, a recent study by Adesina and Mwamba (2018) examined the cyclical nature of capital buffers for 14 African banks covering the period 2004-2014. The findings support the procyclical behavior, like in the preceding studies. That is, there is a positive relationship between the capital buffers and the business cycle, implying that banks increase their capital buffer during economic booms in order to use them during economic recessions. However, there are studies with findings contrary the procyclical view; for example, the studies on Baltic countries (Braslins & Arefjevs, 2014), Turkey (Kontbay-Busun & Kasman, 2015), Pakistan and Tasman (Riaz et al., 2019), Indonesia (Tasman et al., 2019). For some countries, the level of capital buffers required was at the upper bound of 2.5 per cent.

Lastly, the relationship between bank risk and business cycle has also received some attention in the empirical literature. A study by Kontbay-Busun and Kasman (2015) established that the relationship between bank risk and business cyclical was countercyclical during the period 2002 to 2012 for the Turkish economy. However, the recent study by Riaz et al. (2019) showed that business cycle fluctuations had no significant impact on portfolio risk for the Pakistani banks.

It is clear from the theoretical literature that a relationship between the capital buffers, bank risk and economic cycle does exist. However, from the three strands of empirical literature discussed above, there is no consensus about their interrelations. Secondly, there is no study that has been yet conducted on Namibia. Thus, it is of greater importance, given the current economic downturn that Namibia had experienced and continues to experience, that this study addresses this literature gap.

3. Methodology

3.1 Model Specification and Econometric Framework

This study adapted a model by Riaz et al. (2019). The model was modified to suit the Namibian context as follows.

$$CBF_t = \beta_0 + \beta_1 LNA_t + \beta_2 ROA_t + \beta_3 LIQ_t + \beta_4 NPL_t + \mu_t \quad (1)$$

Where, CBF is capital buffer; LNA is log of total assets; ROA is return on assets; LIQ is liquidity; NPL is non-performing loans. The operational definitions for the variables are presented in Table 1 below.

Table 1: Operationalization and measurements of variables

Variables and Proxy	Source	Description
Capital buffer	Rahman et al., (2018), Riaz et al., (2019)	This is capital-to-risk-weighted-assets ratio minus minimum capital ratio. The capital regulation in Namibia, banks have to maintain minimum capital requirement which is 10 per cent of RWA.
Bank size	Bouheni and Rachdi	Natural log of total assets

Profitability (return on assets)	(2015), Riaz et al., (2019) Bouheni and Rachdi (2015), Rahman et al., (2018), Riaz et al., (2019)	Ratio of annual net profit to total assets
Liquidity	Bouheni and Rachdi (2015)	Total loan over total assets
Bank's risk	Raza et al., (2019) Rahman et al., (2018) Riaz et al., (2019)	Ratio of non-performing loans to total assets

Source: (Author's construct, 2022)

The variables of interest are capital buffers and bank's risk. However, the other variables, such as total assets, return on assets and liquidity, are internal bank control variables as also used in the above cited studies. The steps involved are discussed in detail below.

The autoregressive distributed lag (ARDL) modelling technique was used (Pesaran et al., 2001). The choice of this approach, as opposed to the one predominant in the empirical literature, is due to the fact that the data in the public domain is only aggregated. The Central Bank has a clause in agreement with the commercial banks not to share bank-level data because the banking sector is small. It might cause frictions in the sector and deter competition. In addition, this test is suitable for variables with a mixture of order of integration below 2, small sample size (as in this case) and it caters for short and long-term relationships simultaneously.

The unrestricted error correction model (UECM) of ARDL model used to examine the long run and the short run relationship takes the following form.

$$\Delta CBF_t = \gamma_0 + \gamma_1 LNA_{t-1} + \gamma_2 ROA_{t-1} + \gamma_3 LIQ_{t-1} + \gamma_4 NPL_{t-1} + \sum_{i=1}^q \theta_1 \Delta CBF_{t-i} + \sum_{i=1}^q \theta_2 \Delta LNA_{t-i} + \sum_{i=1}^q \theta_3 \Delta ROA_{t-i} + \sum_{i=1}^q \theta_4 \Delta LIQ_{t-i} + \sum_{i=1}^q \theta_5 \Delta NPL_{t-i} + \varepsilon_t \quad (2)$$

The first part of the equation (2) with γ_1 - γ_4 refers to the long run coefficients and the second part with θ_1 - θ_5 refers to the short run coefficients.

The long run relationship among the variables is tested using the F-test for the joint significance of the coefficients of the lagged levels of variables, i.e. (Null hypothesis of no cointegration: $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$) as against (Alternative hypothesis of cointegration: $\gamma_1 \neq \gamma_2 \neq \gamma_3 \neq \gamma_4 \neq 0$). If there is a long-run relationship, the conditional ARDL long run model can be estimated as:

$$\Delta CBF_t - \gamma_0 + \sum_{i=1}^q \gamma_1 CBF_{t-i} + \sum_{i=1}^q \gamma_2 LNA_{t-i} + \sum_{i=1}^q \gamma_3 ROA_{t-i} + \sum_{i=1}^q \gamma_4 LIQ_{t-i} + \sum_{i=1}^q \gamma_5 NPL_{t-i} + \varepsilon_t \quad (3)$$

Finally, the short run dynamic parameters are obtained by estimating an error correction model with the long run estimates using the following specification below:

$$\Delta CBF_t - \tau_0 + \sum_{i=1}^q \tau_1 \Delta CBF_{t-i} + \sum_{i=1}^q \tau_2 \Delta LNA_{t-i} + \sum_{i=1}^q \tau_3 \Delta ROA_{t-i} + \sum_{i=1}^q \tau_4 \Delta LIQ_{t-i} + \sum_{i=1}^q \tau_5 \Delta NPL_{t-i} + \emptyset ECT_{t-i} + \varepsilon_t \quad (4)$$

Where τ_1 - τ_5 refers to the short run dynamic coefficients to equilibrium and \emptyset refers to the speed of adjustment coefficient.

3.2 Data and Data Sources

The period of study is 2001 quarter 1 to 2019 quarter 4. The data for the following variables were collected from the website of the central bank, Bank of Namibia. The variables are capital buffer, total assets, return on assets, liquidity and non-performing loans. All the variables are in ratios with the exception of total assets which was converted to natural logarithms. The choice of the period was influenced by data availability for non-performing loans for the Namibian banking sector.

4. Results

4.1 Descriptive Statistics

Table 2: Descriptive statistics

	CBF	LNA	ROA	LIQ	NPL
Mean	5.133	17.640	3.325	13.368	2.531
Median	5.000	17.680	3.250	10.200	2.650
Maximum	7.500	18.701	4.900	31.800	5.600
Minimum	3.600	16.301	1.500	8.900	1.100

Standard Deviation	0.824	0.689	0.633	6.472	0.974
Skewness	0.776	-0.184	0.430	1.589	0.687
Kurtosis	3.214	1.932	3.601	3.968	3.612
Observations	72	72	72	72	72

Source: (Author's construct, 2022)

Table 2 above shows that the variables are relatively normally distributed. The variable LNA has the highest mean and median values, while the NPL has the lowest mean and median values. The variable LIQ has the highest standard deviation, which suggest some possible high variations, while ROA has the lowest standard deviation.

4.2. Correlation Matrix Analysis

Table 3: Correlation matrix

	CBF	LNA	ROA	LIQ	NPL
CBF	1				
LNA	-0.0295	1			
ROA	-0.0284	-0.3546	1		
LIQ	0.2710	0.7015	-0.0618	1	
NPL	0.4491	-0.7068	0.2528	-0.2042	1

Source: (Author's construct, 2022)

Table 3 shows a negative correlation between assets size and banks' capital buffers. Similarly, there is also an inverse relationship between return on assets and banks' capital buffers. On the contrary, the results show a positive correlation between liquidity and banks' capital buffers as well as between the ratio of non-performing loans to total assets and banks' capital buffers.

4.3 Unit Root

Table 4: Unit root tests: ADF in levels, first and second differences

Variable	Model Specification	ADF		Order of integration
		Levels	First difference	
	Intercept and trend	-3.170**	-8.774**	0
CBF	Intercept	-3.220**	-8.794**	0
	Intercept and trend	-2.971	-11.076**	0
LNA	Intercept	-4.038**	-10.481**	1
	Intercept and trend	-5.363**	-10.708**	0
ROA	Intercept	-5.215**	-10.782**	0
LIQ	Intercept and trend	-0.853	-8.009**	1
	Intercept	0.761	-7.649**	1
NPL	Intercept and trend	-1.539	-7.427**	0
	Intercept	-3.032**	-6.783**	1

Source: (Author's construct, 2022).

Note: * and ** means the rejection of the null hypothesis at 10 and 5 per cent respectively.

The results in Table 4 above reveal the following: First, the variables capital buffers and return on assets are stationary in level, (I (0)). Second, total assets and non-performing loans have a combination order zero with the model specification of intercept and trend. However, they are I (1) when the model specification is intercept only. Third, liquidity is stationary in first difference, (I (1)). The conclusion deduced from Table 4 is that there is a mixture of different order of integration amongst the variables.

4.4 ARDL Bound Testing Cointegration

Table 5: Autoregressive distributed lag (ARDL) Result of Cointegration

Test Statistic	Value	Level of Significance	Lower Critical Value I(0)	Upper Critical Value I(1)	Critical Value
F-statistic	10.47902	10%	2.320	3.232	
		5%	2.725	3.718	
		1%	3.608	4.860	

Source: (Author's construct, 2022)

Table 5 above presents the Bound testing cointegration. In particular, the F-test statistic shows that there is cointegration (long-run relationship). This is because the calculated value of 10.479 is greater than both the lower and upper critical values at all levels of significance, though the findings would still hold if it was greater at least one of the levels of significance. Thus, a conditional ARDL model that includes both the long and short run can be estimated.

Table 6 shows a positive long run relationship between the ratio of NPLs to gross total loans and capital buffers. Thus, the two variables move in the same direction such that when the ratio of NPLs to gross total loans increases, so does the capital buffers. This positive relationship was also found by Kontbay-Busun & Kasman (2015) in their study on Turkey as well as by Belem and Gartner (2016) on Brazil.

4.5 Estimated Long-Run and Short-Run

Table 6: Long-run Estimated Coefficients (dependent variable: capital buffers)

Variable	Coefficient	t-Statistic	Prob.
LNA	0.472	0.175	0.861
ROA	-0.067	-0.519	0.606
LIQ	-0.036	-1.488	0.143
NPL	0.194	0.773	0.443
C	-26.411	-3.980	0.000
Robustness Indicators			
R ²	0.782		
Adjusted R ²	0.708		
F-Statistic	10.575 [0.000]		
D.W Statistic	2.259		
Serial Correlation, F	1.437 [0.248]		
Heteroscedasticity, F	0.279 [0.997]		
Ramsey RESET, F	0.526 [0.472]		
Normality, F	0.802 [0.669]		

Source: (Author's construct, 2022)

This relationship suggests that banks increase capital buffers when there is greater risk as a result of increases in the ratio of NPLs to gross total loans. Similarly, the study also shows a positive relationship between log of total assets and capital buffers. These findings support that of Raza et al. (2019); Noreen et al. (2016) and imply that the bank size, specifically an increase in bank working assets (size) leads to an increase in the buffer capital amount. On the contrary, the study revealed a negative relationship between return on assets and capital buffers as well as between liquidity and capital buffers in Namibia. The latter results support that of Noreen et al. (2016) on Pakistan where a negative relationship between liquidity and capital buffers was found.

Table 7: Short run Estimated Coefficients (dependent variable: capital buffers)

Variable	Coefficient	t-Statistic	Prob.
D(LNA)	0.472	0.248	0.805
D(LNA (-1))	5.587	3.244	0.002**
D(ROA)	-0.067	-0.614	0.542
D(LIQ)	-0.036	-1.721	0.092*
D(NPL)	0.194	0.899	0.373
D(NPL (-1))	-0.627	-2.367	0.022**
ECT (-1)	-0.977	-8.316	0.000**
Robustness Indicators			
R ²	0.640		

Adjusted R ²	0.562
F-Statistic	10.479 [0.010]
D.W Statistic	2.259

Source: (Author's construct, 2022)

Table 7 above presents the results for the short run relationship between capital buffers and the other variables. The table shows that the ratio of NPL to gross total loans positively affects capital buffers similar to the long run relationship results. However, the results show that a lagged variable of the ratio of NPL to gross total loans affects capital buffers negatively and statistically significant. This simply suggests that banks reduce capital buffers in times when there is high risk. For instance, they can extend more credit during economic downturn. This stimulates aggregate demand via consumption and in turn stimulate growth. Furthermore, the variable LNA also positively affects capital buffers though it became statistically significant after lagged once. The positive relationship affirms the findings from the long run model too. Similarly, as it is the case from the long run estimates, the variables ROA and LIQ negatively affect capital buffers; with the latter it is statistically significant.

The short run adjustment process is examined from the ECM coefficient which is (-0.977) and is statistically significant at 5 per cent level of significance. This suggests that it takes about 98 per cent each quarter for capital buffers to correct itself towards equilibrium. Lastly, as it is general practice, the model was checked for stability using various diagnostic tests. The results for normality were confirmed by the Jarque-Bera normality test. The results for autocorrelation confirmed that there is no correlation between the variables. The heteroscedasticity test also confirmed the absence of it in the model and the Ramsey RESET confirmed that the functional form of the model does not suffer from omitted variables. Finally, the adjusted R-squared confirmed the model's ability to explain the total variation in the dependent variable.

4.6 Stability Diagnostic Test

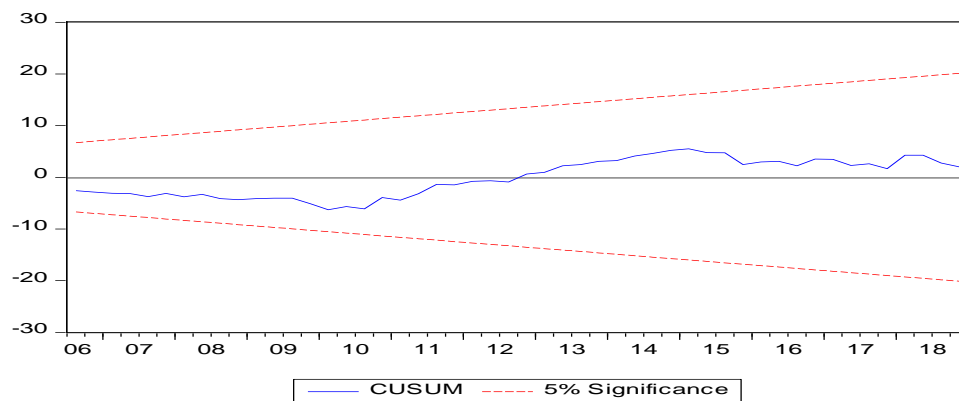


Figure 1: Cumulative Sum (CUSUM)

It is general practice to also ascertain the parameter constancy. Figures 1 and 2 show that the parameters are stable as shown by the cumulative sum (CUSUM) and cumulative sum of square (CUSUMSQ) tests.

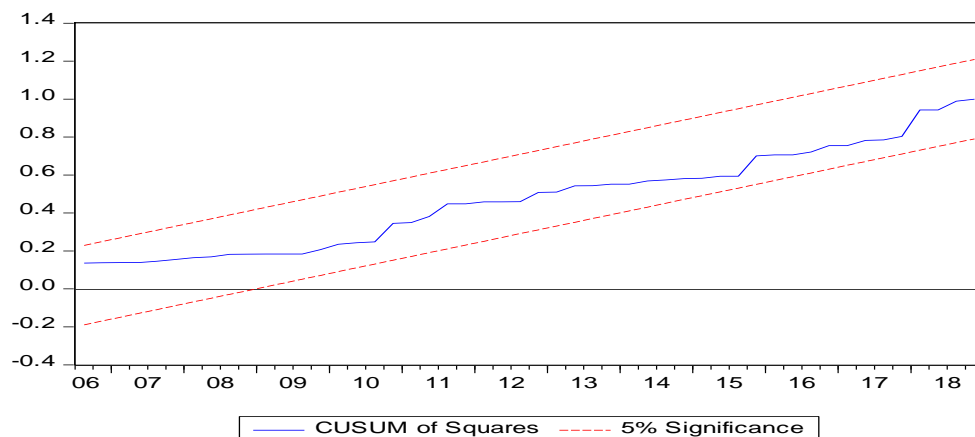


Figure 2: Cumulative Sum of Square (CUSUMSQ)

5. Conclusion and Recommendations

This paper attempted to quantify the effect of bank's risk on capital buffers in Namibia. Using quarterly data between 2001 and 2019, the study employed time series econometric techniques such as unit root, Bound testing cointegration and error correction modelling. The results of the unit root tests revealed a combination of different order of integration. The cointegration test also revealed that the variables do exhibit a long run equilibrium relationship. The results for the long run model show that banks' risk positively affect capital buffer. This is to say, an increase in risks induces banks to increase capital buffers. This could be interpreted as bank's withholding funds that they could lend out in form of credit in order to resuscitate the economy via aggregate demand. Similarly, bank size in form of assets also positively affects capital buffers in the long run. This is usually associated with bank loans. Since most assets of the banks are made up of loans, it simply means more assets implies more capital buffers. On the contrary, return on assets and liquidity negatively affects capital buffers in the long run.

The results from the short run estimates revealed that banks' risk negatively affects capital buffers. This suggests that an increase in the ratio of NPL to gross total loans increases, and capital buffers decreases because banks extend further credit during economic downturn in order to stimulate aggregate demand in the economy in the short run. The effects of return on assets and liquidity remains negative as in the long run. Similarly, the effect of bank size in form of bank assets also positively affects capital buffers in the short run. The policy implication from these findings is that the presence of countercyclical relationship is in support of the transition from Basel II to Basel III to mitigate the procyclical as experienced under Basel II accords as documented in the literature.

5.1 Managerial implication

In practical terms, a positive relationship between bank's risk and capital buffer would imply that an increase in risks induces banks to increase capital buffers. Managers are inclined to cushion the bank's capital buffer by not lending out more in form of credit, which in turn suppress the growth in the economy. Therefore, managers have to rethink about this approach.

5.2 Theoretical implication

This study contributes to literature on banking and finance. The policy implication is that these findings are in support of the transition from Basel II to Basel III to mitigate the procyclical as experienced under Basel II via countercyclical. Although this study sheds some light on the subject matter, future studies should use disaggregated data for the individual commercial banks to see whether or not the outcomes differ.

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Price Prediction for Bitcoin: Does Periodicity Matter?

Gbadebo Adedeji Daniel^{†1}, Akande Joseph Olorunfemi², Adekunle Ahmed Oluwatobi³

¹ Department of Accounting Science, Walter Sisulu University, Mthatha, Eastern Cape, South Africa, <https://orcid.org/0000-0002-1929-3291>

² Department of Accounting Science, Walter Sisulu University, Mthatha, Eastern Cape, South Africa, <https://orcid.org/0000-0001-8445-8905>

³ Department of Accounting Science, Walter Sisulu University, Mthatha, Eastern Cape, South Africa, <https://orcid.org/0000-0003-1603-6705>

ARTICLE INFO	ABSTRACT
<p>Article History</p> <p>Received 18 June 2022 Accepted 1 November 2022</p> <p><i>JEL Classifications</i> C10; G15, G17</p> <p>Keywords: Bitcoin, Diebold-Mariano test, Univariate forecast models, Forecast accuracy</p>	<p>Purpose: A major challenge traders, speculators and investors are grappling with is how to accurately forecast Bitcoin price in the cryptocurrency market. This study is aimed to uncover the best model for the forecasts of Bitcoin price as well as to verify the price series that offers the best predictions performance under different periodicity of datasets.</p> <p>Design/methodology/approach: The study adopts three different data periods to verify whether frequency matters in forecasting Bitcoin price. The Bitcoin price, from 01/01/15 to 11/01/2021, is trained and validated on selected forecast models, including the Naïve, Linear, Exponential Smoothing Model, ARIMA, Neural Network, STL and Holt-Winters filters. Five forecast accuracy measures (RSME, MAE, MPE, MAPE and MASE) are applied to confirm the best performing model. The Diebold-Mariano test is used to compare the forecasts based on the daily price with those based on the weekly and monthly.</p> <p>Findings: Based on the accuracy measures, the results indicate that the Naïve model provides more accurate performance for the daily series, while the linear model outperforms others for the weekly and monthly series. Using the Diebold-Mariano statistics, there is evidence that forecasting Bitcoin price is not sensitive to the data periodicity.</p> <p>Research limitations/implications: The study has a major limitation, which is the shared sentiment to apply actual Bitcoin price series, and not the returns or log transformation for the forecast models. Notably, actual data may sometimes be loud, hence increasing the possibility of over predictions.</p> <p>Originality/value: In forecasting, different approaches have been used, this paper compares outputs of both statistical and machine learning methods in order to arrive at the best option for the Bitcoin price forecasts. Hence, we investigate whether the machine learning tools offer better forecasts in terms of lower error and higher model's accuracy relative to the traditional models.</p>

1. Introduction

There is increasing research on Bitcoin (BTC) in the fields of theoretical and empirical finance. Bitcoin is a cryptocurrency that relies on anonymous peer-to-peer trades via online and social networks interfaces. Its transactions are organised on the Blockchain, an open-source algorithm that uses sophisticated protocol to generate and verify records. Bitcoin shares known attributes with typical financial assets (Baur et al., 2018; Mikhaylov, 2020), and has been exploited as medium of payments as well as accepted in exchange for alternative cryptocurrencies and different national currencies. Bitcoin stands as a speculative asset in times of economic upheavals (Baur et al., 2018), and sometimes perceived as a safe haven and substitute for traditional financial assets (Kliber et al., 2019). During the wave of COVID-19, the price of Bitcoin soared higher relative to conventional assets and commodities (Hung et al., 2020). Bitcoin remains unregulated by any coordinated monetary policy of central banks (Barontini & Holden, 2020).

[†] Corresponding Author: GBADEBO Adedeji Daniel
Email: gbadebo.adedejidanield@gmail.com

However, there are reports on the plan to create Central Bank Digital Currency to regulate Bitcoin and Digital Ledgers (Bofinger & Haas, 2021; IMF, 2020; Auer et al., 2020).

The price of Bitcoin is associated with consistent short- and long- term volatility. The fluctuations in the price is mostly attributed to the limited supply, demand increase, activities of trend chasers and speculations in the bitcoin market. The excessive swings have immersed pressure on users, investors and regulators, leading to increasing interests to forecast its price (Aalborg et al., 2018; Kliber et al., 2019). Studies that focus on forecasting the price of bitcoin use either intraday, daily, weekly or/and monthly series (Bouri et al., 2021; Sitzimis, 2021; Uras et al., 2020). Bouri et al. (2021) employ the functional forecasting approach to examine the intraday trading under the efficient market hypothesis. They provide evidence of profitable trades based on the trading strategies. The bitcoin cumulative intraday return is observed to be heteroscedastic, stationary, non-normal and uncorrelated. Uras et al. (2020) forecast the daily price of bitcoin using different statistical techniques. The authors note that the price appears to be indistinguishable from a random walk process. When the dataset is partitioned into shorter sequences, the evidence confirms the regime hypothesis.

Forecasting the price of Bitcoin has implications for the financial markets. Suitable forecast models offer traders the realistic direction of price, including information on whether to transact on the spot or future markets. The models serve as tools that help investors to circumvent massive losses from sporadic volatility. An accurate forecast model provides the opportunity to increase returns and trading (Munim et al., 2019), since the asset managers would avoid risk by employing the model with least possible error (Kliber et al., 2019). The choice of a forecast model is challenging due to asymmetric information, uncertainties and dynamic behaviours of miners. This study intends to find the best forecast model for Bitcoin price, and on the basis of the different periodicity of datasets, verifies the series that offers the best forecast performance.

We contribute to existing literature in two ways. First, we compare outputs of statistical and machine learning methods in order to arrive at the best option for the Bitcoin price forecast. Forecasting with these approaches have been used in different fields of research (Basher & Sadorsky, 2022; Ye et al., 2022; Chen et al., 2020; Rizwan et al., 2019), including specific application to passenger traffic in coastal shipping (Sitzimis, 2021). We examine whether the machine learning tools offer better forecasts than the traditional models, in terms of lower error and higher accuracy of the model. This becomes necessary in the light of the continuous applications of machine learning approaches which outputs often depict distinct forecast patterns. We train and validate the Bitcoin price series on selected forecasting models as well as compute alternative forecast accuracy to decide the best suitable model. Second, we consider the issue of data frequencies using daily, weekly and monthly series. We check whether the forecast models of Bitcoin price are sensitive to data frequency. The need to test the resilience of periodicity becomes important as the result would offer lead on best choice of dataset to evaluate bitcoin price forecasts, and by extension other alternative cryptocurrencies.

The result shows that for the daily time-series the Naïve model outperforms the others. The evidence based on the Diebold-Mariano statistics indicates that forecasting the Bitcoin price is not sensitive to the data frequency. The rest of the paper is organised as follows. Section two presents a brief trend movement of Bitcoin price. Section three is the material and methodology, where the study summarises the various forecast models and present some measures of forecast accuracy. Section four presents the results including the summary statistics, stationarity tests, forecast models, and the forecast accuracy. Section five is the conclusions.

2. Materials

2.1 Bitcoin Price Trends

Although Bitcoin was reportedly invented in 2009, it first featured on a cryptocurrency exchange on February 6, 2010. Since then, it has witnessed unprecedented and continuous price movements. On March 18, 2013, the US Financial Crimes Enforcement Network issued regulations on virtual currency and legal recognition of bitcoin, and this was believed to motivate the significant increase in bitcoin price from USD149.08 on October 15 to about USD1,242 on November 29. In 2014, there was massive price decline caused by the hacking of the then biggest Bitcoin exchange (Mt. Gox), making the price to rally around USD340.00–USD531.05. The price decline continued and stood at USD434.25 at 2015 end. The Bitcoin splits (hard forks) on August 1, 2017, marks monumental strides in BTC price rallies, with massive run up (buy orders), pressuring the price to reach an all-time high of USD19,783.06 on December 17, 2017.

The increase could not be sustained, therefore the price dropped to USD13,412.44 by January 1, 2018. Figure 1 shows the daily price from July 1, 2018, to June 30, 2021. The price experience massive run-up, resistance, reversals, different supports and consolidations. The price dropped to USD6,300 on October 31, 2018, and dipped further below USD3,300 by December 7, 2018. The price started above USD3,700 in 2019, and stood at USD7,200 by year end. In November 2020, the price rallied above USD18,000, regaining its losses from 2017 peak. The price later surpassed its previous peaks, crossed above USD40,000 and landed on a remarkable daily average all-time high of about USD 64,863.31 on April 14, 2021. The price has fallen about 40% to USD40,044.54 in June 2021.

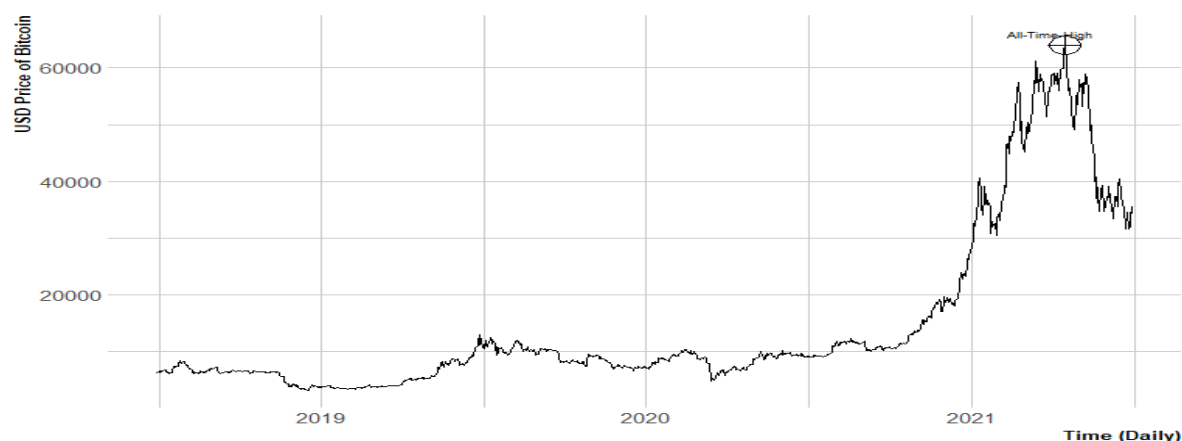


Figure 1: Daily Price of BTC in USD

Source: (Author's construct, 2023)

2.2 Empirical Highlights

Time-series literature recommends model-based and univariate-based methods for forecasting volatile assets. The first approach predicts bitcoin price as dependent on some factors (Gbadebo et al., 2021; Jaquart et al., 2021; Koutmos & Payne, 2020; Liang et al., 2020). Gbadebo et al. (2021) employ the Autoregressive Distributed Lag (ARDL) to verify how Bitcoin price volatility responds to cryptocurrency capitalisation, equity index, trading volume and Google search. The study confirms the existence of long run cointegration and conclude that market fundamentals drive the volatility of price than information demand. Jaquart et al. (2021) use artificial neural network (ANN), random forests (RF) and long short-term memory (LSTM) to analyse how blockchain, technical, sentiment and asset returns explain Bitcoin price forecast. The quantile result shows the long-short trading strategy creates about 39% returns. Liang et al. (2020) apply the GARCH-MIDAS model to investigate competing index predictors. They provide that the Chicago Board Options Exchange (CBOE)'s gold volatility index exhibits strongest predictability for the BTC price volatility relative to the CBOE volatility index, google trends, global economic policy uncertainty and geopolitical risk. Koutmos (2020) uses a Markov regime-switching model to show that asset pricing factors such as stock price, interest rate and exchange rates are the main determinants of Bitcoin price.

The application of the model-based approach has notable limitations, including depending on prior assumptions made about the series' distribution. As noted, (Aalborg et al., 2018), predicting Bitcoin price on the basis of these fundamental indicators is still ambiguous. Hence, the second approach based on univariate times-series would be more suitable for forecasting the Bitcoin price. Caporale et al. (2018) establish the existence of correlation amongst past and present values of the BTC price. Many studies (Basher & Sadorsky, 2022; Ye et al., 2022; Aygün & Günay Kabakçı, 2021; Chen et al., 2020; Munim et al., 2019; Adcock & Gradojevic, 2019; Mallqui & Fernandes, 2019; Rizwan et al., 2019; McNally et al., 2018) confirm the robustness of the univariate approach. Ye et al. (2022) apply an ensemble machine learning model to forecast Bitcoin's next prices. They combine both the LSTM and Gated Recurrent Unit (GRU) with stacking ensemble system and use sentiment indexes, technical indicators to forecast Bitcoin prices, during September 2017 to January 2021. The results indicate that the near-real time forecast exhibit better performance MAE of 88.74%. Basher and Sadorsky (2022) use random forests and bagging classifiers and the logit models to predict Bitcoin prices. The accuracy for the random forests and the bagging classifiers range above 85% for 10 to 20 days prediction and between 75% and 80% for the 5-day forecasts. They conclude that the random forests predict the Bitcoin price with much accuracy than the logit models. Aygün and Günay Kabakçı (2021) explore the MA, ARIMA as well as machine learnings (ANN, RNN) and convolutional neural network (CNN) of Bitcoin price predictions. The RNN offers better performance relative to other methods. Hamayel and Owda (2021) employ three machine learning methods (LSTM, bi-LSTM and GRU) to predict Bitcoin, Litecoin, and Ethereum. The GRU model show the smallest MAPE and RMSE, outperforming other algorithms.

Chen et al. (2020) compare support vector machine (SVM) and long short-term memory (LSTM) and showed that, for the next day BTC price, the SVM provides a higher accuracy of 65.3% classification. Demir et al. (2019) predict the price of Bitcoin using methods such as long LSTM, NB, as well as the nearest neighbour technique. These methods achieved prediction accuracy between 81.2% and 97.2%. Mallqui and Fernandes (2019) employ artificial neural network (ANN) and support vector machines (SVM) algorithms in regression models to forecast the maximum, minimum and closing Bitcoin prices. He concludes that SVM algorithm outperformed the ANN with lowest mean absolute percentage error (MAPE) of 1.58%. McNally et al. (2018) employ the Bayesian recurrent neural network (RNN) and LSTM to forecast the daily movement in the price of Bitcoin. The LSTM achieve a high performance with the classification accuracy of 52% and a root mean squared error (RMSE) of 8%. Munim et al.

(2019) employ an autoregressive integrated moving average (ARIMA) and a neural network autoregression (NNAR). They split the data into two training-sets, and for the first training-set, the NNAR outperforms the ARIMA, while for the second, the ARIMA outperforms the NNAR. Velankar et al. (2018) use the generalized linear (GLM) model and Bayesian regression to forecast the daily average price change signals and uncover a prediction accuracy rates of 51% with the GLM. Adcock and Gradojevic (2019) use the feed-forward neural networks (FNN), GARCH-M, ARIMAX, random walk and multiple regression to predict prices. They examine how 50-200 days moving averages (MA) of bitcoin volume and VIX affect its prices, which shows little significance on its forecasts. The FNN indicates the highest accurate density and point forecast relative to other models.

3. Methodology

3.1 Forecast models and predictive accuracy

Organizational the study employs univariate-based forecast models. Each model is evaluated based on the accuracy of its predictions *vis-à-vis* actual data. We adopt five methods (RMSE, MAE, MPE, MAPE and MASE) to assess the accuracy of the forecast methods. To avoid the over-fitting problem, we trim the time-series into two sets: Training and validation (test) sets. We scrutinise the data behaviour as well as consider the data frequency and forecast horizon in deciding the length for the validation periods (Hyndman & Athanasopoulos, 2021). We select a forecast horizon which does not exceed the validation periods to arrive at training-set (01\01\15–30\06\19) and validation-set (01\07\19–11\01\2021) for the daily time series. The weekly has training (01\01\15 – 27\06\19) and validation (28\06\19 – 11\01\2021), while the monthly is trained on (01\01\15 – 01\07\19) and validated on (01\08\19 – 11\01\2021). The forecast errors of the models in Table 1a are used to compute the accuracy measures. Table 1b presents the various measures of forecast accuracy.

3.2. The Data

We employ Bitcoin price from the Finance.yahoo's official website. The database stores historical data on Bitcoin price from the real time price on the CoinMarketCap Exchange. The daily data obtained, spanning 01\01\15 to 11\01\21, reports the opening, lowest, highest and closing prices. We apply the closing price in line with previous studies (Uras et al., 2020; Chen et al., 2020; Munim et al., 2019). Previous studies apply daily data (Uras et al., 2020; Chen et al., 2020), while some others employ weekly (Othman et al., 2020) and/or monthly (Ramadhani et al., 2018) series for forecasting bitcoin price. Because we aim to verify whether periodicity matters in the performance of the forecast, we use three different datasets.

In this paper, we do not apply log transformation for the different series used. We share the sentiments to verify the Bitcoin price forecasts in its original form because there are downside to forecasting security prices or returns in logarithm (Hudson & Gregoriou, 2010) or other transformation forms (Meucci & Quant, 2010). As noted, (Hudson & Gregoriou, 2010), the mean of a set of random variables computed using logarithmic is often less than the mean computed from the simple set, specifically by an amount dependent on the variance of the set. In effects, when the log series are applied, *ceteris paribus*, higher variance will inevitably reduce the mean price or returns.

Table 1a: Summary forecast models

Model	Explanation	Model Algorithms (Equations)	References
Naïve Model (NAÏVE)	Naïve model uses observations of the previous period to forecast the next. The method takes the last observation as the forecast. Let y_t ($t = 1, 2 \dots, T$) denotes Bitcoin price, y_T as actual value of the last observation. Divide y_t to: training set ($t = 1, 2, \dots, n$) and validation set [$t = n + 1, n + 2, \dots, n + v (=T)$]; e_t is forecast error.	$\hat{y}_{T+h T} = y_T$ $e_t = y_t - \hat{y}_{T+h T}$ $(\hat{y}_{T+h T}) = h\text{-step forecast.}$	Stenqvist & Lonno (2017)
Linear Trend (LINEAR)	Linear trend creates forecasts values been a generalisation of y_t as a time-trends. The trend-line approach is used if the y_t series exhibits steady increase or decrease overtime and an error term (ε_t). A polynomial function for y_t depends on Trend (T), trend square (T^2) and a drift (ψ_0).	$y_t = \psi_0 + \psi_1 T + \psi_2 T^2 + \varepsilon_t;$ $\hat{y}_{T+h T} = y_t + e_t;$ $\varepsilon_t = \hat{y}_{T+h T} - y_t$	Bisht & Agarwa (2017) Ostertagová (2012)
Exponential	The ETS creates forecast weighted	$y_t = \mu_t + \beta_t t + S_{t,p} + \varepsilon_t$	Liantoni &

Smooth Model (ETS)	averages with the recent observations more weighted than distant ones when determining the forecasts. The weights (ϕ_j) diminish exponentially [$\phi_j = \phi^j; -1 < \phi < 1$ ($j = 1, 2, \dots, m$)]. The 3 (time-varying) components: mean (μ_t), slope (β_t) and seasonality, $S_{t,p}$ ($p = 1, 2, \dots, P$) for p seasons, $\forall t, \beta_t = 0$ & $\forall p, S_{t,p} = 0$. The smoothing starts by computing at $t = 1$. T_t is smoothed slope that estimates β_t , L_t is smoothed level that estimates μ_t ; S_t is smoothed seasonality that estimates $S_{t,p}$. Initial estimates smoothing-states: $t = 0: L_0, T_0$, and $S_{0,1}, \dots, S_{0,P}$ use for the Smoothing equations (L_t, S_t and T_t).	$L_t = \alpha(y_t - S_{t-p}) + (1 - \alpha)L_{t-1}$ $S_t = \delta(y_t - L_t) + (1 - \delta)S_{t-p}$ $\hat{y}_{T+h T} = L_t + S_{t-p+h}$ Invertible region: $\max(-P\alpha, 0) < \delta(1 - \alpha) < 2 - \alpha.$	Agusti (2020) Olvera-Juarez & Huerta-Manzanilla (2019)
ARIMA Model	ARIMA has an autoregressive [AR(p)], a moving average [MA(q)] and an order of integration components, where d is the number (#) of differencing required to attain a stationary [ARMA (p, q)] model and q is the order of the MA component. μ is the intercept (drift time-series, which is often zero), y_{t-i} ($i = 1, \dots, p$) is previous time series periods until lag p , θ_i is the parameter for y_{t-i} , ε_t is the error term in time t , ε_{t-j} is the error term of all previous periods until lag q and δ_j ($j = 1, \dots, q$) is the parameter for ε_{t-j} .	General ARIMA (p, d, q) model is: $\Delta y_t = \mu + \sum_{i=1}^p \theta_i \Delta y_{t-i} + \sum_{j=1}^q \delta_j \varepsilon_{t-j} + \varepsilon_t$ $\Delta y_t = y_t - y_{t-1}$	Munim <i>et al.</i> (2019) McNally <i>et al.</i> (2018) Bakar & Rosbi (2017)
NNAR Model	NNAR is a sophisticated neurone-like elements assembled in layers. While simple NNAR is analogues to linear regression model with inputs (predictors) and output (dependent variable), the complex NNAR is nonlinear. y_j is actual state of output unit j in the input-output; X_j is the input – vector; α_j is constant for node j , $W_{i,j}$ is weight-vector from input node i to output node j , and m is # of inputs. The parameters $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ & $W_{1,1}, \dots, W_{4,3}$, are 'learned' from training data. Before training, we restrict $W_{i,j}$ & set as 0.1. If y_j is transformed via sigmoid squashing, we get $s(y)$, where β, k, c and τ are constants. The learning is reduced to a minimum error with repeated changing of $W_{i,j}$ by an amount (δ_j) proportional to $\partial E / \partial W_{i,j}$. \hat{y}_j is	NNAR error back-propagation algorithms: $y_j = \alpha_j + \sum_i^m W_{ij} X_j$ $s(y) = \beta + \frac{k}{1 + e^{\tau y}}$ $E = \frac{1}{2} \sum_j^n (y_j - \hat{y}_j)^2$ $\Delta W_j(t + 1) = \lambda \delta_j y_j$ $\delta_j = (\hat{y}_j - y_j) f'(y_j)$ $(y_{t-1}, \dots, y_{t-p}, y_{t-m}, \dots, y_{t-pm})$	Chen <i>et al.</i> (2020) Munim <i>et al.</i> (2019) Mallqui & Fernandes (2019) McNally <i>et al.</i> (2018)

	desired state and the learning rate, λ is kept constant. To forecast with the NNAR, the lagged values of the univariate series is used as inputs. A feed-forward NNAR with one hidden layer is denoted $NNAR(p, k)$ or $NNAR(p, P, k)_m$, where p is lag-length or p last observations used as inputs, k is the # of nodes (neurons) in the hidden layer and p is # of seasonality.		
STL Model	STL adopts a non-parametric algorithm that iterates loess smoother to refine y_t into 3 components. y_t consists of a trend (T_t), a seasonality (S_t) and an irregularity (I_t). The STL assumes S_t has the same cycle periodically. The cycle adopts a spectral analysis which shows the characteristics of oscillations of different wavelengths. The spectrum of a process y_t with an autocorrelation function (ω_τ) where, $\sum_{\tau=1}^n \omega_\tau < \infty$ is denoted $y(\omega_\tau)$. STL protocol set for T_t smoothing parameter is: $t.window \geq \lceil \frac{1.5 * frequency}{1 - \frac{15}{s.window}} \rceil$ (must be odd integer ≥ 7).	$y_t = T_t + S_t + I_t$ (additive split) $y_t = T_t * S_t * I_t$ (multiplicative split) $y(\omega_\tau) = \omega + 2 \sum_{\tau=1}^{\infty} \omega_\tau \cos(2\pi\omega\tau)$	Hyndman & Athanasopoulos (2021)
Holt-Winters Model (HWM)	HWM is a typical deterministic model with a trend, seasonality and residuals. HWM computes a smooth series $\hat{y}_{T+h t}$ with recursively updating equations that allow for the iterative computation of forecasts based additive or multiplicative protocols. The additive algorithm is criticised not to generate best estimates for time-series level and seasonality. We adopted a multiplicative algorithm, which assumes the seasonal effect is proportional to a time change. The level (p_t), trend (b_t), and seasonality (s_t) which depend on the smoothing parameters $\alpha, \beta, \gamma \in [0, 1]$. The forecast h -step at time $T + h$ given data up to time t , and the constant k is the seasonality. The estimation of α, β , and γ is through the minimisation of randomly chosen errors. To estimate $\alpha, \beta, \gamma \in [0, 1]$, a robust smoothing process centred on M -estimation uses: $\hat{y}_{T+h t} = \arg \min \sum_{t=1}^T e_t^2$. We presented forecast for HWM with trend but no seasonal component $HWM(\gamma[False])$.	$y_t = \mu_t + \beta_t t + S_{t,p} + \varepsilon_t$ $p_t = \alpha \frac{y_t}{s_{t-k}} + (1 - \alpha)(p_{t-1} + b_{t-1})$ $b_t = \beta^*(p_t - p_{t-1}) + (1 - \beta^*)b_{t-1}$ $s_t = \gamma \frac{y_t}{p_{t-1} + b_{t-1}} + (1 - \gamma)s_{t-k}$ $\hat{y}_{T+h t} = (p_t + hb_t)s_{t-k+h_k^*}$	Brügner (2017) Kuang <i>et al.</i> (2016)

Table 1b: Predictive accuracy measures

Accuracy measure	Accuracy scale/computation
RMSE	$\left(\frac{1}{m} \sum_{t=1}^m (e_t)^2\right)^{1/2}$
MAE	$\frac{1}{m} \sum_{t=1}^m e_t $
MPE	$\frac{100}{m} \sum_{t=1}^m \frac{e_t}{y_t}$
MAPE	$\frac{100}{m} \sum_{t=1}^m \left \frac{e_t}{y_t} \right $
MASE	$\frac{1}{m} \sum_{t=n+1}^{n+m} e_t \bigg/ \frac{1}{n-1} \sum_{t=1}^n y_{t-1} - y_t $

Mean Absolute Error (MAE) gives the magnitude of the average absolute error in all periods. Root Mean Square Error (RMSE) and Mean Absolute Percent Error (MAPE) provide a percentage score of how forecasts deviate from the actual values. MASE compares predictive model performance to the Naïve forecast on the training set.

Source: (Author's construct, 2023)

3.3. Estimation Process

We adopt a static forecast approach for estimation. The approach ensures the univariate variable's actual value in previous periods is employed to estimate each step forecast. We follow the standard process of time-series forecasting, identifying the time series into training sample (observed datasets) and validation samples (observed datasets). We model the data with training samples and evaluated the forecast performance with validation samples. We combine the series, train the model on the full observed data and use the performance to forecast future prices. The data are trained on all forecast models with training datasets. We select a test period that mimics the predictive horizon for the future forecasts' valuation of performance.

We adopt library (*forecast*) and library (*fpp2*) in RStudio. We apply the *stl* (time series, *s.window*="periodic") function to decompose y_t by obtaining T_t using loess and calculate S_t (and I_t) as $y_t - T_t$. By default setting for the *s.window* parameter, the function *stl()* assumes S_t follows the same cycle yearly. To ensure equal-spaced data, the study resolves the problem of non-multiple integer periodicity in infra-monthly high-frequency data by following Hyndman and Athanasopoulos (2021). The study periods have two leap-year-days (29\02\2016 and 29\02\2020). We set the frequency at 365.25 for daily series with the function *ts(dataset, start = c(2015, 1), frequency = 365.25)*.

In the computation of the HWM, the study omits the seasonal component then set the function *Holt-Winters* (dataset, gamma = false) which allows for 365 - long vector of the initial seasonal pattern as its argument. We could not do otherwise since the Holt-Winters function (dataset, gamma = "integer") requires frequency to be multiple of the length of observations for the forecast to be computed in the next cycle. The *ets()* functions ignore the seasonality for infra monthly data with a frequency greater than 24 during computation. The function *auto.arima()* library in R selects and returns best ARIMA model through AIC, AICc or BIC¹ values. The order of the ARIMA model was selected through automatic iteration. The *nnetar()* function fits an $NNAR(p, P, k)_m$ model. If the values of p and P are not defined, the lag is selected automatically according to the AIC for a linear $AR(p)$ model.

Before we proceed to forecasting, we complete three diagnostic tests - Box-Ljung (BL) autocorrelation test, Box-Pierce (BP) x-square residual test and the Jarque-Bera (JB) normality test to determine the validity of the forecast models. The LB test is a portmanteau test for the "overall" randomness based on some lags, with the test null that the residuals from the forecast model (fitted) have no autocorrelation. The BP test with a test statistic (Q_m) verifies whether the series is pure white noise. The Diebold-Mariano (DM) test compares two forecast models. It determines whether one forecast model is more accurate than the other.

4. The Results

4.1. Data statistics

Table 2 presents the deterministic statistical properties for the price of Bitcoin for each periodicity, including their training-set and validation-set partitions. The table shows that all series are asymmetrically distributed with positive skewness. For the training and validation sets, the daily dataset with 1.299 and 3.038 degree of skewness, respectively appears more skewed compared to other frequencies. The training samples appear to be mesokurtic (moderately peaked), while the others are leptokurtic (high peaked) for all frequencies. The outliers are more on the validation

¹ Autocorrelation Function (ACF); Partial Autocorrelation Function (PACF); Akaike Information Criterion (AIC); corrected Akaike Information Criterion (AICc); Bayesian information criterion (BIC).

samples. We reject the normality null for all the data partitions with a highly significant Jarque-Bera test. The Bitcoin price plot (Figure 1a) supposes the data may not be stationary. The non-stationarity would be confirmed with the unit root test.

Figure 1a – 1f represent plots for the daily Bitcoin price (full data), the training sets, validation periods, the first difference, log daily price and the log-difference. The weekly (Figure W1 – W6) and monthly (Figure M1 – M6) plots are presented in the appendix. The plots replicate same shape with the daily plots, except that the infra-monthly plots show more volatility, outliers, and breaks. The daily series presents multiple, non-integer periodicities associated with high volatility with microstructure effect (Urquhart, 2018), while the monthly series appear smoother with less clustering. All observed series are chaotic with spiky protrusions. The log-transformed series appear with smoother striations.

4.2. Time-series decomposition

Figure 2a – 2c present the decomposition of daily, weekly and monthly. We apply the `stl()` function to decompose the observed data (topmost graph) into key time-series components. The function segregates the deterministic ('trend' and 'seasonal') and stochastic ('random') components of the Bitcoin price series. We apply the daily, weekly and monthly seasonal window. The trend component reflects the long-term progression (upward movement) of the series over-time, while the remainder (residual) is convergence with mean reversing. The seasonality is oscillatory with repetitive pattern over-time. In the daily series, the trend appeared unchanged and stable around January 2015 to February 2017. After these periods, the frequency and amplitude of the cycle upsurge over time. With the Loess framework, Bitcoin price shows exponential trends upward with additive seasonality. The residuals are quite random, particularly exhibiting high variability around late 2017 during the first remarkable price peak. Table 3 presents the summary statistics of the STL decomposition.

Table 2: Data deterministic statistics

Statistics	Daily			Weekly			Monthly		
	Training	Validation	Full	Training	Validation	Full	Training	Validation	Full
Mean	3365.8	10923.4	5290.4	3368.0	11299.3	5401.1	3346.8	12797.2	5901.0
Median	1184.6	9641.5	4141.9	1166.0	9607.2	4255.5	1140.8	9696.3	4411.3
Maximum	19513.0	40402.0	40402.0	17517.1	38255.1	38255.1	13742.3	34662.5	34662.5
Minimum	194.3	4987.6	194.3	194.3	5791.6	194.3	231.5	7285.0	231.5
Std. Dev.	3726.3	4978.3	5243.7	3713.6	5779.0	5545.8	3667.2	7983.7	6649.0
Skewness	1.3	3.0	1.6	1.2	2.9	1.9	1.1	2.0	2.2
Kurtosis	4.4	14.2	8.3	4.1	12.0	9.8	3.3	5.7	9.6
JB(Stat)	595.8	3787.8	3568.8	73.4	384.9	795.1	11.0	19.2	195.9
JB (p-value)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0041	0.0001	0.0000
# of Obs.	1642	561	2203	235	81	316	54	20	74

Note: JB: Jarque-Bera, # of Obs.: Number of Observations

Source: (Author's construct, 2023)

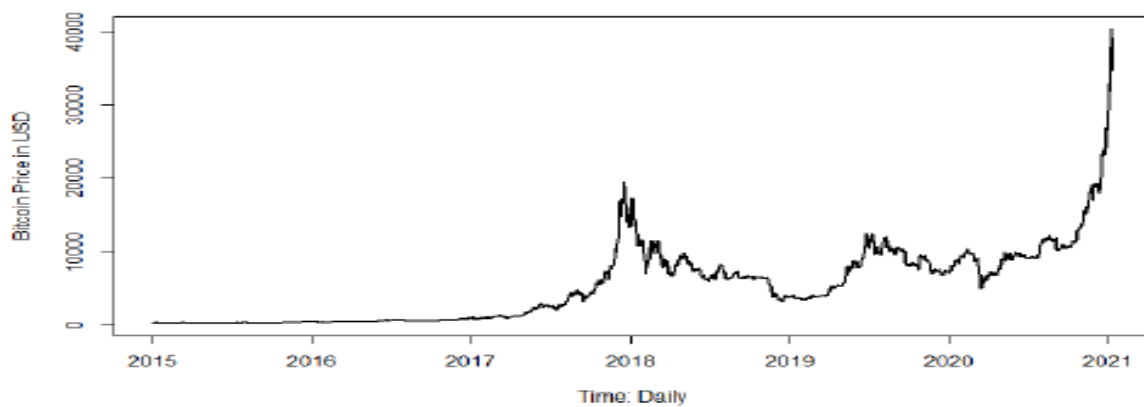


Figure 1a: Daily Bitcoin Price in USD (01-15-21 to 11-01-21)

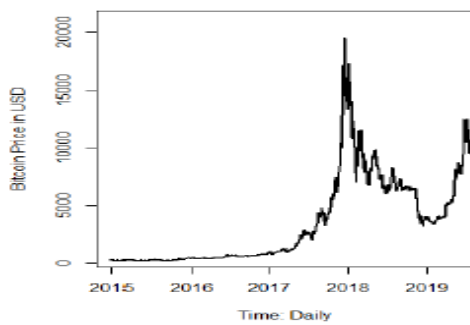


Figure 1b: Bitcoin Price (01-01-15 to 30-06-19)

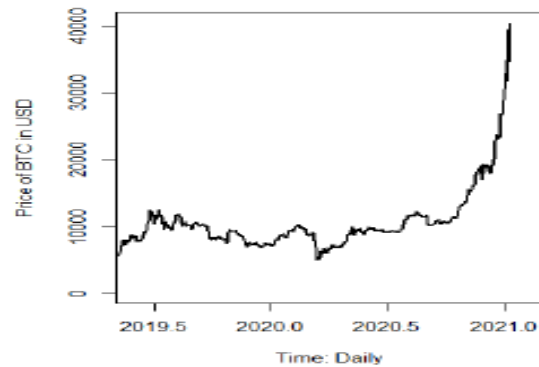


Figure 1c: Bitcoin Price (01-07-19 to 11-01-21)

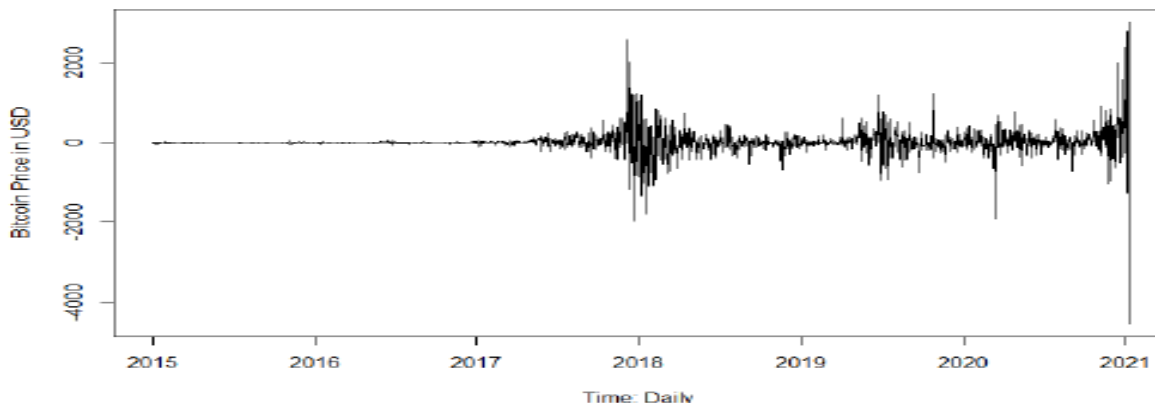


Figure 1d: Bitcoin Price Difference (01-15-21 to 11-01-21)



Figure 1e: Bitcoin Price (Log)

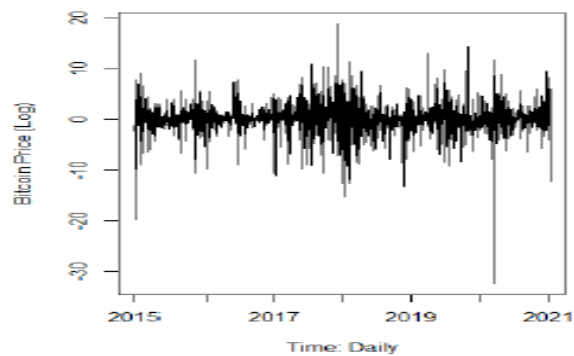


Figure 1f: Bitcoin Price (Log difference)

4.3. Stationarity test

The stationarity result (Table 4) shows that the ADF test accepts the null of non-stationarity, with $\tau_\mu > ADF_\alpha$ in all the test equations. The ERS statistics for the validation-set (daily) and training-set (weekly) appear stationary but this was refuted when we add the linear trend, hence the ERS nulls are accepted at 1%. The KPSS rejects the test's null of stationarity at 1%. The results confirm non-stationarity for the training, validation and combined data, for all series. The first difference tests are all stationary and significant at 1%, except for the validation-set for monthly series at 5% (no trend) and 10% (linear trend). Bitcoin price for each periodicity is clearly, $I(1)$, and indistinguishable from a random walk.

4.4. Training the Bitcoin price data

We train the daily series for 1642 days (01\01\15 – 30\06\19) and evaluate the models for a validation period of 561 days (01\07\19 – 11\01\2021). The weekly data was trained for 235 weeks (01\01\15 – 27\06\19) and validated for 81 weeks (28\06\19 – 11\01\2021). The monthly series was trained for 54 months (01\01\15 – 01\07\19) and validated for 20 months (01\08\19 – 11\01\2021). For the daily series, the Naïve forecast produces a residual

standard error of 220.38. The linear model and its trend coefficients are significant with the model p -value of approximately zero. The ETS (M,Ad,N) parameters reported are $[\alpha(0.9999), \beta(0.1887), \Psi(0.8)]$, with Initial states $[(a = 308.66, b = -3.43)]$, and σ (sigma) = 0.03. The ARIMA (auto) uses the lowest AIC to select an ARIMA (2, 1, 0) while considering the specification's stationarity test. There was an average of about 20 different network specifications in the neural network.

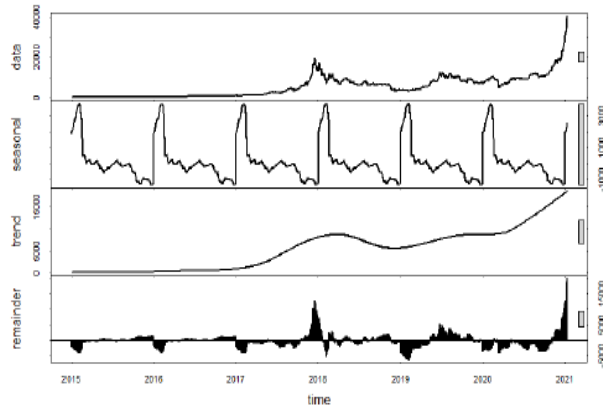


Figure 2a: STL decomposition of Bitcoin price

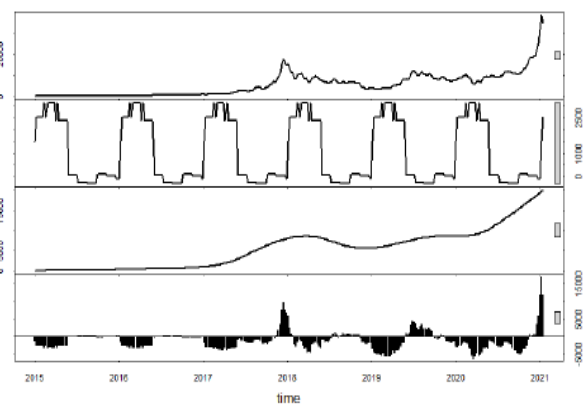


Figure 2b: STL decomposition of Bitcoin price (weekly)

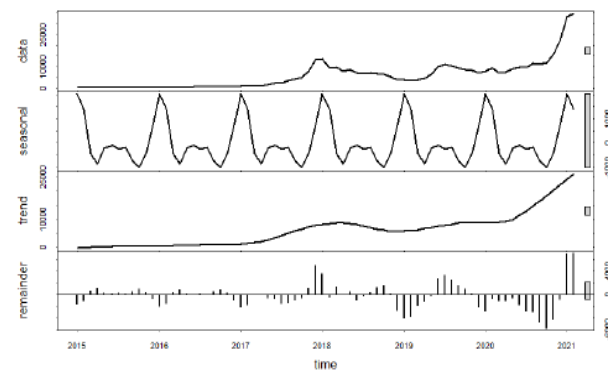


Figure 2c: STL decomposition of Bitcoin price (monthly)

Table 3: STL decomposition statistics

Statistics	Daily			Weekly			Monthly		
	Seasonal	Trend	Random	Seasonal	Trend	Random	Seasonal	Trend	Random
Min.	-1340.10	278.04	-6274.98	-278.75	32.08	-6207.36	-1007.97	-266.34	-5626.28
1st Qu.	-544.17	630.86	-1389.27	-92.46	637.03	-2789.17	-428.82	676.24	-1265.32
Median	-116.95	5863.26	-112.48	113.96	5886.31	-1371.57	-210.77	5980.26	24.66
Mean	102.54	5370.82	-182.97	1034.65	5483.00	-1116.60	46.12	6019.32	-164.46
3rd Qu.	138.14	8493.48	711.59	2435.79	8471.77	142.39	724.53	8505.58	613.74
Max.	3777.14	18402.07	19826.54	3153.41	20124.91	16912.48	2022.29	26240.30	7031.75
IQR	682.30	7862.60	2100.90	2528.00	7835.00	2932.00	1153.00	7829.00	1879.00
IQR%	8.40	97.00	25.90	31.20	96.60	36.10	13.80	93.70	22.50

Qu.: Quartile. IQR: Interquartile range IQR%: Percentage IQR.

Source: (Author's construct, 2023)

Table 4: Stationarity test

	Level (y_t)						Difference (Δy_t)						
	τ_μ	ADF_α	τ_τ	ERS_α	τ_η	$KPSS_\alpha$	τ_μ	ADF_α	Prob.	τ_τ	ERS_α	τ_η	$KPSS_\alpha$
Daily													
Training	-0.84	-3.43	-0.05	-2.57	3.14	0.74	-26.5	-3.43	0.00	-26.50	-2.57	0.11	0.74
Validation	3.19	-3.34	3.45	-2.57	1.39	0.74	-7.78	-3.44	0.00	-2.13	-1.94	1.18	0.74
Full	-1.43	-3.97	-2.57	3.11	4.27	0.74	-28.3	-3.43	0.00	-11.41	-2.57	0.52	0.74
Training ^a	2.18	3.31	-1.86	-3.48	0.34	0.22	-26.5	-3.96	0.00	-8.32	-3.48	0.07	0.22
Validation ^a	2.97	-3.97	0.42	-3.48	0.53	0.22	-8.47	-3.97	0.00	-3.48	-0.12	0.19	0.22
Full ^a	1.16	-3.96	-0.77	-3.48	0.19	0.22	-28.4	-3.96	0.00	-9.09	-3.48	0.52	0.74
Weekly													
Training	-0.73	-3.46	3.25	-2.58	0.97	0.74	-7.67	-3.46	0.00	-7.61	-2.58	0.10	0.74
Validation	1.92	-3.51	1.30	-2.59	0.66	0.74	-5.99	-3.51	0.00	-5.98	-2.59	0.68	0.74
Full	0.99	-3.45	1.53	-2.57	1.70	0.74	-5.94	-3.45	0.00	-5.35	-2.57	0.41	0.74
Training ^a	-2.36	-4.00	-0.42	-3.50	0.30	0.22	-7.70	-4.00	0.00	-5.48	-3.46	0.06	0.22
Validation ^a	1.13	-4.08	-1.01	-3.65	0.25	0.22	-6.72	-4.08	0.00	-5.53	-3.65	0.14	0.22
Full ^a	-0.75	-3.99	-1.22	-3.47	0.19	0.22	-6.18	-3.99	0.00	-4.14	-3.47	0.15	0.22
Monthly													
Training	-1.24	-3.56	1.38	-2.64	0.57	0.74	-4.68	-3.56	0.00	-4.74	-2.61	0.11	0.74
Validation	2.29	-3.81	-0.63	-2.69	0.48	0.74	-4.53	-3.81	0.00	-2.58	-2.69	0.45	0.74
Full	0.34	-3.52	1.46	-2.60	1.01	0.74	-4.99	-3.52	0.00	-4.97	-2.60	0.35	0.74
Training ^a	-2.59	-4.14	2.23	-3.77	0.19	0.22	-4.66	-4.14	0.00	-4.70	-3.76	0.08	0.22
Validation ^a	0.76	-4.50	-2.08	-3.77	0.18	0.22	-3.49	-4.50	0.07***	-3.27	-3.77	0.14	0.22
Full ^a	-1.60	-4.09	-1.16	-3.67	0.09	0.22	-5.27	-4.09	0.00	-5.33	-3.69	0.13	0.22

ADF_α : MacKinnon one-sided p-values; Elliott-Rothenberg-Stock (ERS_α); Kwiatkowski-Phillips-Schmidt-Shin ($KPSS_\alpha$).

ADF Null (H_0): Nonstationary; DF-GLS Null (H_0): Non-stationary; KPSS, Null (H_0): Stationary

^aTest has intercept with linear trend, others are with no (time) trend; the Critical Value(C.V.) reported are at 1%;

** stationarity at 5%; *** Stationary at 10%.

Source: (Author's construct, 2023)

The NNAR (4, 1, 3) with σ^2 estimated as 49270 was selected based on test-sample. We estimate Holt-Winters model with trend and without seasonal component, which accommodates for non-multiple of the number of observations. The smoothing parameters and coefficients obtained are $[\alpha(1), \beta(0.022), \gamma(\text{False})]$ and $[a(11636.46)$ and $b(87.49)]$, respectively. We estimate the weekly and monthly sets and compared the forecast performance with our daily counterparts. Next, we apply these models to predict the price of BTC for the validations periods to shed light on performance. Figure 3a–3g shows the time-series plots of actual and predicted values during training and validation periods for the daily series, while Figure 4a–4g and Figure 5a–5g (appendix) show same for weekly and monthly datasets. Table D.1 (appendix) presents a 40-day (01/07/19–09/08/19) summary of predictions, as well as forecast errors (absolute and percentage) in the validation periods for the daily price of Bitcoin. The table presents the average point forecast, 80%, and 95% intervals for each forecasting method. A cursory look at the table indicates the result favours the Naïve forecast performance – which presents data-frame of lower errors – relative to other predictive measures. The forecast accuracy measures are employed to make appropriate judgment on the best forecast model.

4.5. Forecast accuracy

Table 5 presents the training sample and validation sample forecast performance evaluated with the forecast accuracy measures. When we trained the daily series on each forecast model except for the MPE, four of the accuracy measures [RMSE, MAE, MASE and MAPE] showed that the Naïve model performed better than other predictive models. The Naïve model has the least values for the various measures as indicated [with asterisk *] in Table 5. With the weekly series and using the RMSE, MAE and MAPE as evaluation benchmarks, the Naïve method still outperformed other models. However, the MPE support that the linear model is best and the MAPE indicates that the Exponential

smoothing model outperformed others. The monthly series also confirmed the superiority of the Naïve method over others as three of the accuracy measures when we trained with the monthly data shows Naïve method has the lowest forecast error. Turning to the validation samples evaluation, the results supported the HWM's superiority over others for the daily sample, except for the MAPE and MASE measures.

Table 6 presents the result of the DM tests. We compare the accuracy of the forecast performance from two different models under same data frequency. The result is similar to reports in Table 5. Comparing the Naïve model (F_1) to another forecast models (F_2) for each of the data frequency, we confirm that the Naïve model is more accurate in forecasting the test sample price ($p < 5\%$), which is not surprising since a better forecast for BTC price is its last previous price. For all the data frequency, the DM tests confirm the ARIMA superiority over the NNAR in the test-sample periods (Munim *et al.*, 2019). We complete some residuals diagnostic tests to verify the validity of the forecast models (see Table D.2 in the appendix). The Lbox (Q^*) statistics suggest the presence of autocorrelation, while the Qm (χ^2) test indicates the occurrence of conditional heteroscedasticity, except for the Naïve model.

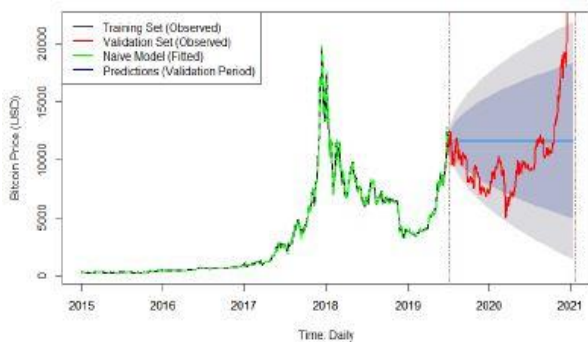


Figure 3a: Prediction in the validation period (Naïve model)

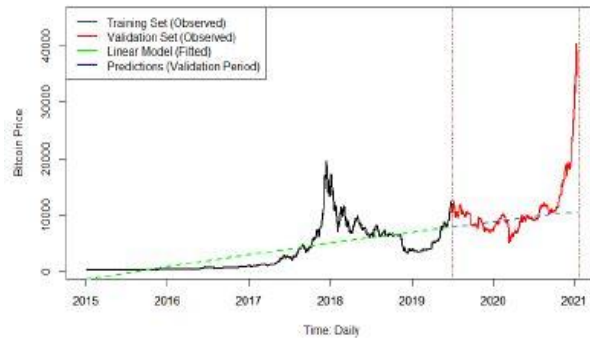


Figure 3b: Prediction in the validation period (Linear model)

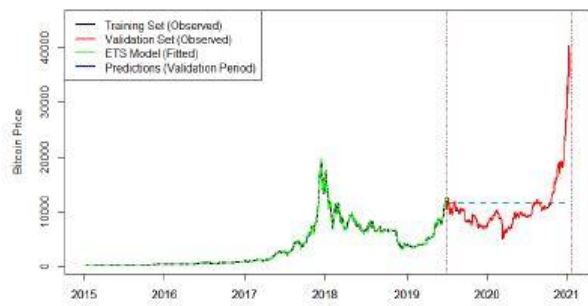


Figure 3c: Prediction in the validation period (ETS model)

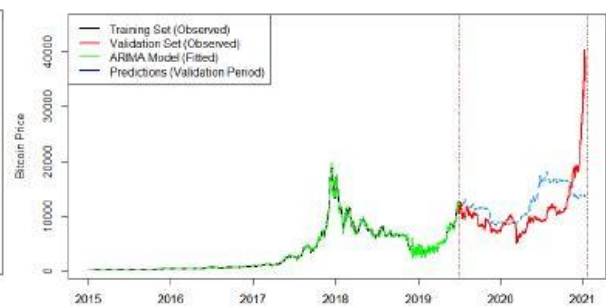


Figure 3d: Prediction in the validation period (ARIMA model)

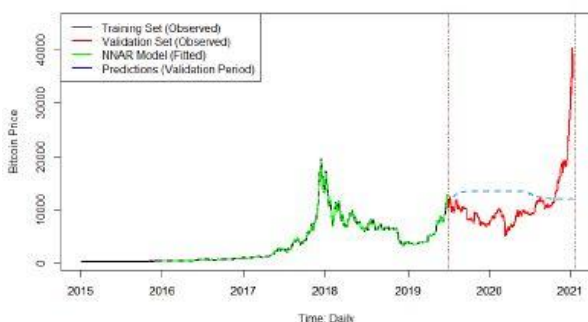


Figure 3e: Prediction in the validation period (NNAR model)

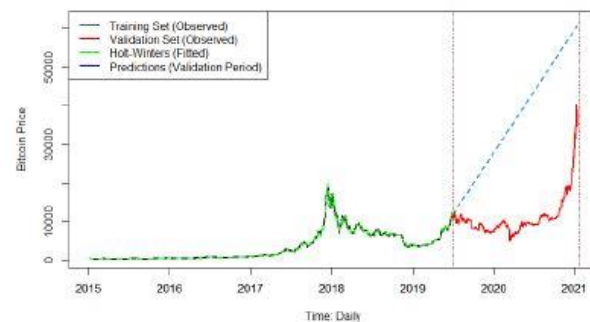


Figure 3f: Prediction in the validation period: (Holt-Winters model)

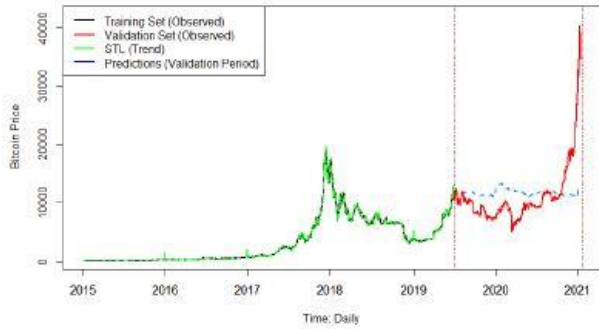


Figure 3g: Prediction in the validation period (STL-trend)

4.6. Future forecasts for Bitcoin price

We predict the out-of-sample forecasts of the price for the daily periodicity. We combine the training and validation periods and estimate the forecast models on the full data. A total of 2203, 316 and 75 observations are applied for the daily, weekly and monthly series, respectively. The forecast periods follow directly behind the closing of validation periods starting January 12, 2021. The forecast horizon does not exceed the validation periods. We compare the prediction intervals for different models with different levels and decreasing certainty for varying future depicted by the prediction cone (Figure 6a–6g). The figures show the future forecasts for daily Bitcoin price. Table 7 shows 40-days forecasts and future returns (percentage) for the daily price. The value of the last observed day Bitcoin price (\$34,662.48) was used to calculate the static percent increase for the 40 days (12-01-2021– 20-02-2021).

4.7. Is the Bitcoin price forecast sensitive to the choice of data frequency?

We check whether forecasting the price is sensitive to the data choice. The forecast accuracy result obtained is presented in Table 5. For the weekly series, the linear is superior except for the MPE measures, indicating that the HWM outperforms others. In contrast, for the monthly series, the Naïve method outperforms others except for the MPE that shows the ETS is superior. Comparing with daily series, we conclude that frequency matters in forecasting the Bitcoin price series. Overall, the results of the model comparison tests (Table 6) establish that irrespective of the data frequency, the Naïve model is superior and more accurately predicts the price than others. The DM test is sufficient to submit that forecasting the price is not sensitive to the periodicity.

Table 5: Training-sample and validation-sample forecast performance

Forecast Methods	Training					Validation				
	RMSE (\$)	MAE (\$)	MPE (%)	MAPE (%)	MASE (I)	RMSE (\$)	MAE (\$)	MPE (%)	MAPE (%)	MASE (I)
Daily										
NAIVE	204.83*	87.451*	0.145	1.990*	0.03*	4993.43	3184.47	-16.70	28.75	0.94*
LINEAR	2591.4	1767.5	-16.96**	144.0	0.52	4900.70	2540.04	6.593	18.46*	0.95
ESM	215.07	91.292	0.112	2.050	0.03	5011.04	3276.57	-18.42	29.98	0.96
ARIMA	281.55	133.23	0.071	2.790	0.04	5032.21	3607.43	-23.14	32.08	1.06
NNAR	221.97	108.72	0.432	2.230	0.03	5769.21	4492.51	-33.83	44.52	1.32
STL	228.75	107.80	0.280	5.430	0.03	5069.70	3424.49	-19.72	31.56	1.01
HWM	220.31	92.241	0.121	2.060	0.03	28056.3**	25298.4**	-250.8**	250.8	7.44
Weekly										
NAIVE	479.74*	242.38*	0.752	6.610	0.07*	6049.76	4670.00	-33.58	44.00	1.38
LINEAR	2552.6	1748.6	-16.43**	142.0	0.52	5643.41**	2786.10**	6.512	18.48**	0.83**
ESM	589.89	270.74	0.310	6.280*	0.08	25218.2	22953.2	-225.5	225.5	6.80
ARIMA	695.08	368.39	0.472	8.350	0.11	6246.72	4975.46	-39.21	45.76	1.48
NNAR	532.01	302.70	-1.68	6.660	0.09	8335.71	6578.63	-28.16	58.98	1.95
STL	615.87	328.21	1.701	19.90	0.10	7649.24	6951.81	-65.75	72.24	2.06
HWM	591.38	267.27	0.940	6.240	0.08	36425.1	32865.4	-316.4*	316.4	9.74
Monthly										
NAIVE	1210.5	636.69*	4.061	13.54*	0.19*	8010.76	4502.49	-4.320	27.71	1.32

LINEAR	2464.3	1784.5	-16.28*	141.6	0.52	7991.66**	4103.34**	12.17	21.19*	1.20**
ESM	1316.8	657.19	2.703	14.00	0.20	19550.7	17722.8	-167.3**	167.3	5.20**
ARIMA	1764.3	965.43	1.971	18.32	0.28	7451.33	4754.54	-13.73	31.21	1.39
NNAR	1083.7*	694.43	-6.571	20.85	0.20	9501.09	5416.06	28.82	29.33	1.59
STL	1218.9	787.38	10.33	61.87	0.23	8411.28	4519.36	10.44	24.68	1.32
HWM	1305.3	648.19	9.363	16.52	0.19	7822.63	4268.52	-1.334	25.29	1.25

MASE is an index (I) which compares a chosen model predictive performance (for instance, the MPE) to the naïve forecast on the training set. The index value less than 1 indicates that the compared model has a lower average error than naïve forecasts (in the training period). If the index value is higher than 1, it indicates poor performance relative to (training period) naïve forecasts.

* Naïve method better.

** Other model outperformed naïve.

Source: (Author's construct, 2023)

Table 6: The Diebold-Mariano (DM) test for test-sample

$F_1(\downarrow)$ $F_2(\rightarrow)$	Table 6: The Diebold-Mariano (DM) test for test sample											
	ETS		LINEAR		ARIMA		NNAR		STL		HWM	
	Daily											
NAIVE	7.6 2	(4.34 $\times 10^{-14}$)	-14.02	(2.20 $\times 10^{-16}$)	- 6.42	(1.75 $\times 10^{-10}$)	- 7.52	(3.97 $\times 10^{-14}$)	- 8.97	(2.20 $\times 10^{-16}$)	- 7.38	(2.52 $\times 10^{-13}$)
ETS			-14.03	(2.20 $\times 10^{-16}$)	- 9.94	(2.20 $\times 10^{-12}$)	- 9.03	(2.20 $\times 10^{-16}$)	- 7.88	(6.48 $\times 10^{-15}$)	- 7.38	(2.48 $\times 10^{-13}$)
LINEAR					13.9 5	(2.20 $\times 10^{-16}$)	15.7 4	(2.20 $\times 10^{-16}$)	- 8.14	(7.79 $\times 10^{-16}$)	- 5.52	(3.96 $\times 10^{-8}$)
ARIMA							15.7 4	(2.20 $\times 10^{-16}$)	- 8.14	(7.95 $\times 10^{-16}$)	- 7.38	(2.51 $\times 10^{-13}$)
NNAR									- 8.96	(2.20 $\times 10^{-16}$)	- 7.38	(2.56 $\times 10^{-13}$)
STL											- 7.38	(2.47 $\times 10^{-13}$)
	Weekly											
NAIVE	4.2 1	(3.66 $\times 10^{-14}$)	-5.42	(1.45 $\times 10^{-7}$)	- 3.44	(0.0007)	2.59 (0.0052)		4.20	(3.83 $\times 10^{-5}$)	- 1.41	(0.1612)
ETS			-5.50	(1.02 $\times 10^{-7}$)	- 3.44	(0.0006)	- 5.53	(8.53 $\times 10^{-8}$)	4.20	(3.83 $\times 10^{-5}$)	- 3.65	(0.0003)
LINEAR				(1.02 $\times 10^{-7}$)	(4.07 $\times 10^{-7}$)	5.21	(4.67 $\times 10^{-9}$)	6.09	4.19	(3.91 $\times 10^{-5}$)	5.42	(1.46 $\times 10^{-7}$)
ARIMA							3.93 (0.0001)		4.20	(3.85 $\times 10^{-5}$)	5.42	(1.46 $\times 10^{-7}$)
NNAR									4.20	(3.81 $\times 10^{-5}$)	- 1.85	(0.0661)
STL											4.20	(3.87 $\times 10^{-5}$)
	Monthly											
NAIVE	2.7 7	(7.79 $\times 10^{-5}$)	-2.76	(0.0079)	- 2.13	(0.0374)	2.05 (0.0453)		2.22 (0.0308)		- 2.24	(0.0495)
ETS			-3.16	(0.0002)	- 3.25	(0.0020)	- 3.34	(0.0015)	2.06 (0.0443)		- 2.37	(0.0214)
LINEAR					1.93	(0.0589)	2.66 (0.0031)		2.08 (0.0429)		2.41	(0.0194)
ARIMA							2.05 (0.0453)		2.04 (0.0464)		- 2.35	(0.0150)
NNAR									- 2.35	(0.0225)	- 2.26	(0.0281)
STL											- 2.32	(0.0193)

DM test compares two forecast models $[F_1, F_2]$. It shows whether (F_1) is more accurate than model (F_2) . The test is based on the loss differentials, $d_t = L(e_{1,t}) - L(e_{2,t})$. $H_0: E[d_t] = 0$ (F_1 is same as F_2) and $H_1: E[d_t] \neq 0$. Assume $e_{j,t} = \hat{y}_{T+h|T} - y_t$, sample mean loss differential $\bar{d} = T^{-1} \sum_{t=1}^T [L(e_{1,t}) - L(e_{2,t})]$ and DM statistic $(DM_\alpha) = \bar{d} / \sqrt{T^{-1} [2\pi \hat{f}(0)]} \rightarrow d \sim n(0, 1)$, where $2\pi \hat{f}(0)$ is a consistent estimator of the asymptotic variance, $\sqrt{T} \bar{d}$. Since DM_α converge to a normal distribution, H_0 is rejected at 5% if $|DM_\tau| > 1.96$, but cannot be rejected, if $|DM_\tau| \leq 1.96$. Probability $(p) < 0.05$ indicates that F_1 is better. Figure in the parenthesis indicate p -value, others are DM_τ . Source: (Author's construct, 2023)

Date	Future forecasts for daily price							Percent increase from previous daily price (daily returns)									
	NAIVE	ETS	LINEAR	ARIMA	NNAR	STL	HWM	F	ETS	LIN.	ARIMA	NNAR	STL	HWM			
12/1/21	35228.94	34255.39	31581.96	32919.03	33815.88	34104.02	34994.37	1.63	-1.17	-8.89	-5.03	-2.44	-1.61	0.96			
13/1/21	35463.58	33929.33	31604.16	33131.23	35419.89	33656.85	35326.26	0.67	-0.95	0.07	0.64	4.74	-1.31	0.95			
14/1/21	35643.62	33668.48	31626.97	33636.11	37077.15	33299.11	35658.15	0.51	-0.77	0.07	1.52	4.68	-1.06	0.94			
15/1/21	35795.41	33439.80	31648.59	33477.27	37614.15	33199.07	35990.04	0.43	-0.62	0.07	-0.47	1.45	-0.30	0.93			
16/1/21	35929.13	33292.86	31670.81	33507.14	39106.08	32952.50	36321.92	0.37	-0.50	0.07	0.09	3.97	-0.74	0.92			
17/1/21	36050.03	33159.30	31693.04	33642.51	37202.53	32769.34	36653.81	0.34	-0.40	0.07	0.40	-4.87	-0.56	0.91			
18/1/21	36161.20	33052.45	31715.27	33628.89	35287.09	32622.81	36985.70	0.31	-0.32	0.07	-0.04	-5.15	-0.45	0.91			
19/1/21	36264.68	32966.98	31737.51	33390.49	36993.15	32487.96	37317.59	0.29	-0.26	0.07	-0.71	4.83	-0.41	0.90			
20/1/21	36361.87	32898.59	31759.77	33373.33	39825.69	32518.07	37649.48	0.27	-0.21	0.07	-0.05	7.66	0.09	0.89			
21/1/21	36453.79	32843.89	31782.02	33444.91	40162.88	32443.04	37981.37	0.25	-0.17	0.07	0.21	8.85	-0.23	0.88			
22/1/21	36541.22	32800.12	31804.29	33287.60	40610.10	32383.02	38313.26	0.24	-0.13	0.07	-0.47	1.11	-0.18	0.87			
23/1/21	36624.76	32765.11	31826.56	33115.95	41528.43	32458.89	38645.15	0.23	-0.11	0.07	-0.52	2.26	0.23	0.87			
24/1/21	36704.89	32737.10	31848.84	33118.52	38265.45	32614.55	38977.03	0.22	-0.09	0.07	0.01	-7.86	0.48	0.86			
25/1/21	36781.99	32714.69	31871.12	33151.03	38425.51	32583.82	39308.92	0.21	-0.07	0.07	0.10	0.42	-0.09	0.85			
26/1/21	36856.38	32696.76	31893.42	33444.37	39854.05	32559.24	39640.81	0.20	-0.05	0.07	0.88	3.72	-0.08	0.84			
27/1/21	36928.33	32682.42	31915.72	33780.11	40211.47	32733.64	39972.70	0.20	-0.04	0.07	1.00	0.90	0.54	0.84			
28/1/21	36998.07	32670.95	31938.03	34064.71	39049.12	32855.33	40304.59	0.19	-0.04	0.07	0.84	-2.89	0.37	0.83			
29/1/21	37065.78	32661.77	31960.34	34086.94	41098.66	32842.75	40636.48	0.18	-0.03	0.07	0.07	5.25	-0.04	0.82			
30/1/21	37131.63	32654.43	31982.66	34140.52	41179.34	32832.68	40968.37	0.18	-0.02	0.07	0.16	0.20	-0.03	0.82			
31/1/21	37195.78	32648.55	32004.99	34086.56	38673.54	32962.05	41300.25	0.17	-0.02	0.07	-0.16	-6.09	0.39	0.81			
01/2/21	37258.34	32643.85	32027.33	34079.07	39086.09	32997.06	41632.14	0.17	-0.01	0.07	-0.02	1.07	0.11	0.80			
02/2/21	37319.43	32640.09	32049.67	34098.15	40936.35	32991.91	41964.03	0.16	-0.01	0.07	0.06	4.73	-0.02	0.80			
03/2/21	37379.14	32637.09	32072.02	33965.90	40562.40	32987.78	42295.92	0.16	-0.01	0.07	-0.39	-0.91	-0.01	0.79			
04/2/21	37437.57	32634.68	32094.38	34069.73	40843.01	33025.94	42627.81	0.16	-0.01	0.07	0.31	0.69	0.12	0.78			
05/2/21	37494.79	32632.76	32116.75	34381.01	41788.23	33034.37	42959.70	0.15	-0.01	0.07	0.91	2.31	0.03	0.78			
06/2/21	37550.88	32631.22	32139.12	34482.50	40013.55	33032.26	43291.59	0.15	0.00	0.07	0.30	-4.25	-0.01	0.77			
07/2/21	37605.91	32629.98	32161.50	34503.17	39536.39	33030.57	43623.47	0.15	0.00	0.07	0.06	-1.19	-0.01	0.77			
08/2/21	37659.92	32629.00	32183.89	34668.17	40953.30	33040.29	43955.36	0.14	0.00	0.07	0.48	3.58	0.03	0.76			
09/2/21	37712.97	32628.21	32206.28	34742.22	40916.83	32941.29	44287.25	0.14	0.00	0.07	0.21	-0.09	-0.30	0.76			
10/2/21	37765.12	32627.58	32228.68	34652.02	39794.47	32940.42	44619.14	0.14	0.00	0.07	-0.26	-2.74	0.00	0.75			
11/2/21	37816.41	32627.07	32251.09	34986.39	41303.55	32939.73	44951.03	0.14	0.00	0.07	0.96	3.79	0.00	0.74			
12/2/21	37866.88	32626.67	32273.50	35019.51	41776.01	32841.25	45282.92	0.13	0.00	0.07	0.09	1.14	-0.30	0.74			
13/2/21	37916.56	32626.35	32292.93	34939.95	40103.16	32445.97	45614.81	0.13	0.00	0.07	-0.23	-4.00	-1.20	0.73			
14/2/21	37965.50	32626.09	32318.36	34896.87	41021.09	32445.61	45946.70	0.13	0.00	0.07	-0.12	2.29	0.00	0.73			
15/2/21	38013.72	32625.88	32340.79	34608.77	42168.37	32445.33	46278.58	0.13	0.00	0.07	-0.83	2.80	0.00	0.72			
16/2/21	38061.26	32625.72	32363.24	34514.60	40757.48	32050.26	46610.47	0.13	0.00	0.07	-0.27	-3.35	-1.22	0.72			
17/2/21	38108.14	32625.58	32385.69	34548.77	40939.21	32050.08	46942.36	0.12	0.00	0.07	0.10	0.45	0.00	0.71			
18/2/21	38154.39	32625.48	32408.15	34702.49	42078.47	30888.73	47274.25	0.12	0.00	0.07	0.44	2.78	-3.62	0.71			
19/2/21	38200.04	32625.39	32430.61	34313.74	40755.05	30888.61	47606.14	0.12	0.00	0.07	-1.12	-3.15	0.00	0.70			
20/2/21	38245.10	32625.33	32453.09	34361.77	40330.67	29727.31	47938.03	0.12	0.00	0.07	0.14	-1.04	-3.76	0.70			

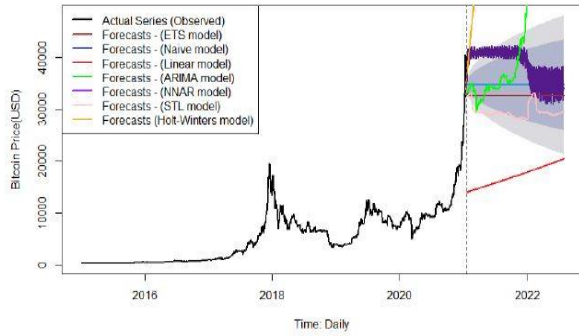


Figure 6a: Daily Bitcoin future forecasts (Naïve model)

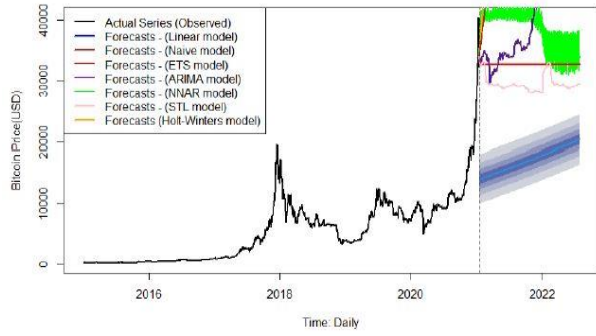


Figure 6b: Daily Bitcoin future forecasts (Linear model)

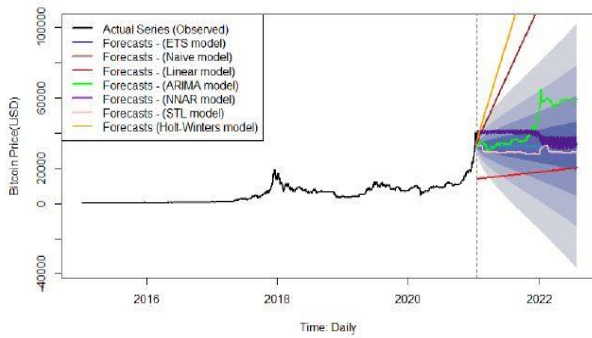


Figure 6c: Daily Bitcoin future forecasts (ETS model)

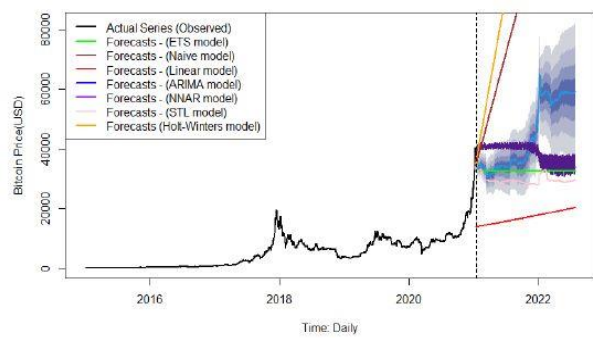


Figure 6d: Daily Bitcoin future forecasts (ARIMA model)

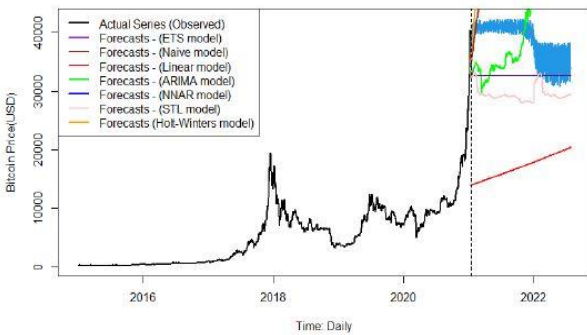


Figure 6e: Daily Bitcoin future forecasts (NNAR model)

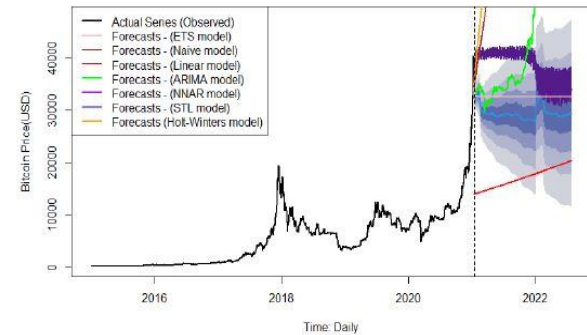


Figure 6f: Daily Bitcoin future forecasts (STL)

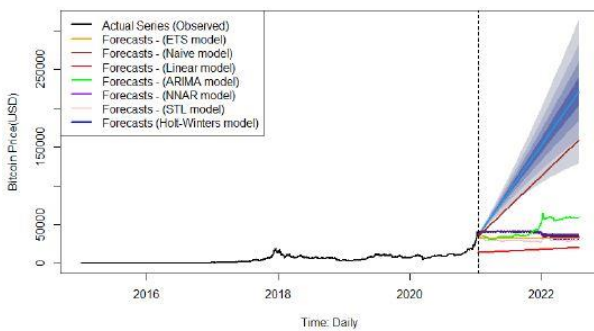


Figure 6f: Daily Bitcoin future forecasts (Holt-Winter model)

5. Conclusions

The study aims to compare the outcome of statistical and machine learning models, and to verify how the different periodicity of the Bitcoin price series, including daily, weekly and monthly, performs in the forecast. We completed forecast models using the Naïve, Linear, Exponential Smoothing, ARIMA, Neural Network, STL and Holt-Winters filters, and apply the standard measures to evaluate the forecast accuracy. The results indicate that the Naïve model provides more accurate performance for the daily series, while the linear model outperforms for both weekly and monthly series. Using the DM statistics to check the forecast equality of the model, the evidence shows that forecasting Bitcoin price is not sensitive to the data periodicity.

The findings have some significant implications. First, because of information asymmetric, increasing economic uncertainties and other markets dynamics, adopting forecast models to predict the directions of Bitcoin price is vital. Second, since Bitcoin has now attracted different stakeholders, including institutional investors, the forecast models of Bitcoin price would serve as guides to make informed decisions in the cryptocurrency markets. Accurate prediction would offer warnings signals to investors, traders and other users in order to circumvent or at least minimize potential-risks due to excessive volatility. Third, the models have implications to drive asset allocations. Asset managers may want to avoid losses by adopting the least error model to predict the likely direction and value of bitcoin price. In periods where volatility is excessive, and the outcome of forecast models becomes sensitive to changes in the training sets, managers may switch funds to invest in financial market assets.

The study has two major limitations: first, for the different periodicity, we apply only the actual price series, and not the returns. Since actual data is usually noisy and may increase the risk of over predictions, we suppose future research can consider other transformation, involving using logarithm or even log-returns. Second, we do not consider the issue of intraday trading. By so doing, we have ignored to convert the models to a trading strategy, which can be compared to possible Monte Carlo of trading strategies where the buy/sell decisions are completely random.

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Appendix

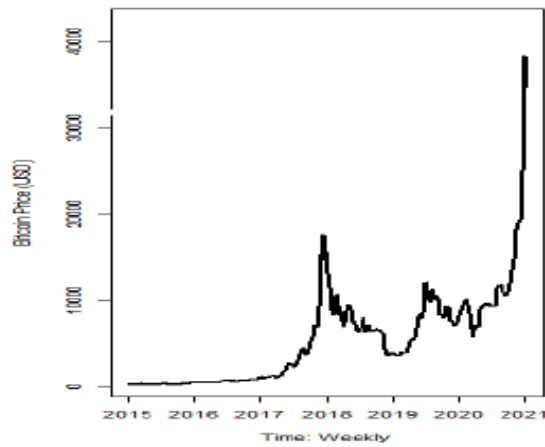


Figure W1: Weekly BTC Price in USD

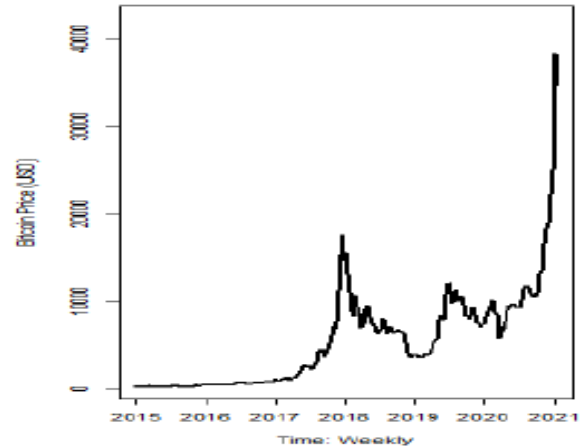


Figure W2: BTC Price 01-01-15 – 27-06-19

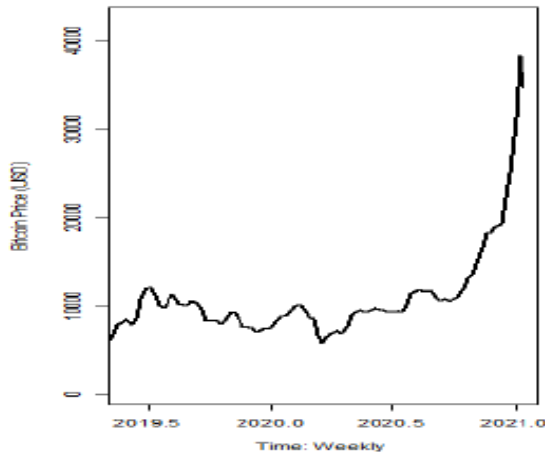


Figure W3: BTC Price 04-07-19 – 11-01-21

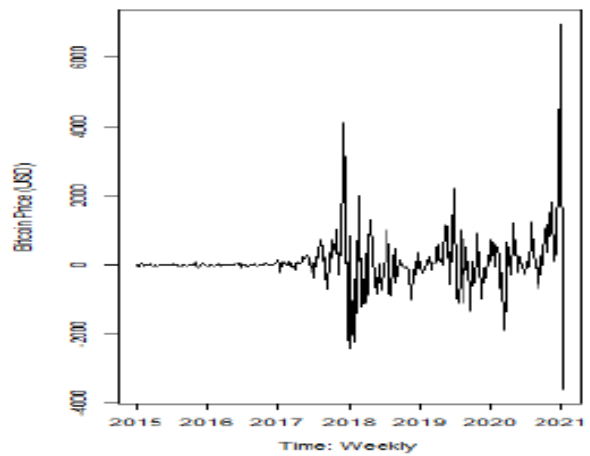


Figure W4: Weekly BTC Price (Difference)

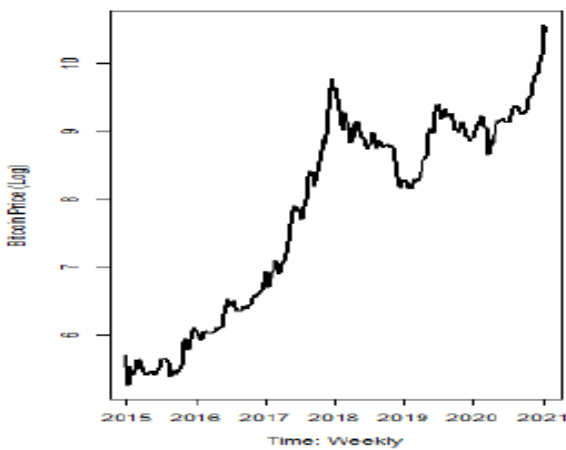


Figure W5: Log of Weekly BTC Price

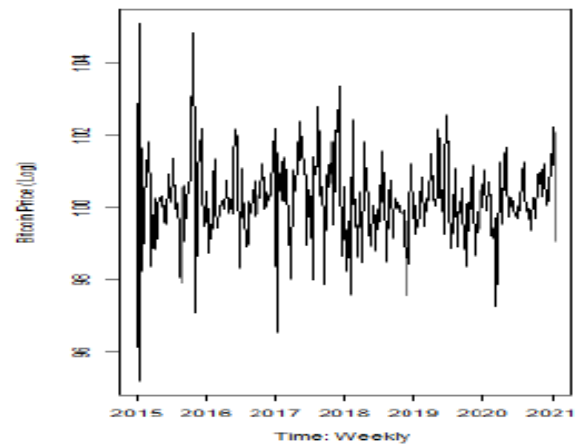


Figure W6: Log of Weekly BTC Price (Difference)

¹Corresponding Author: GBADEBO Adedeji Daniel
Email: gbadebo.adedejidanield@gmail.com

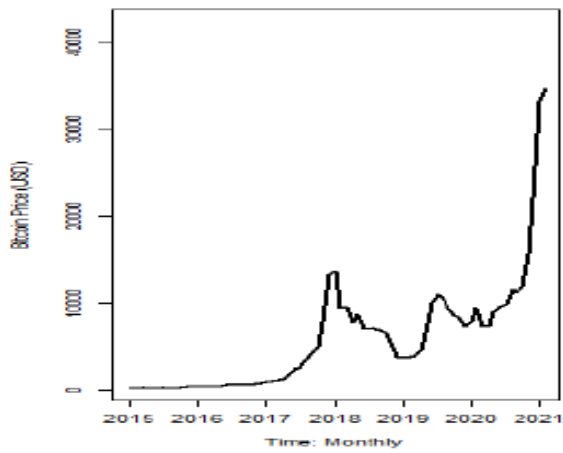


Figure M1: Monthly BTC Price in USD

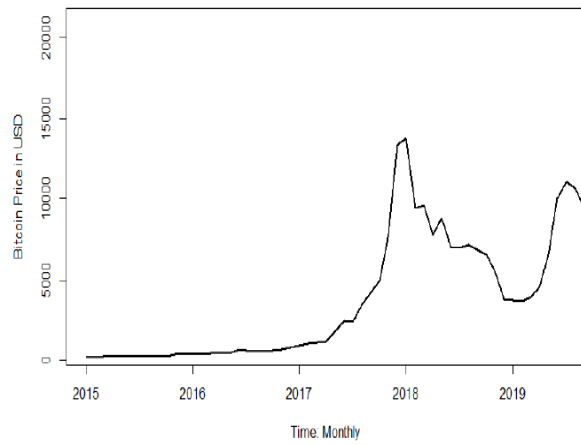


Figure M2: BTC Price 01-15 – 06-19

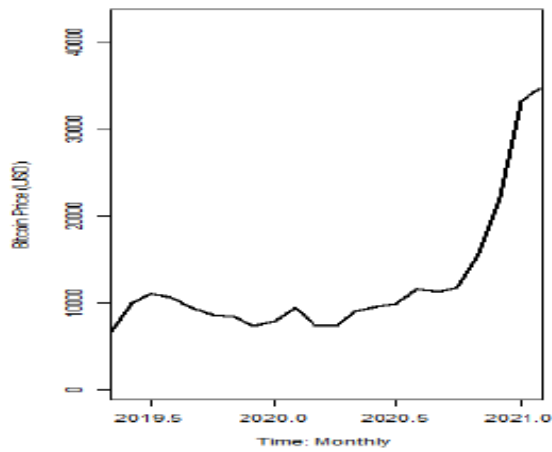


Figure M3: Monthly BTC Price 07-19 – 01-21

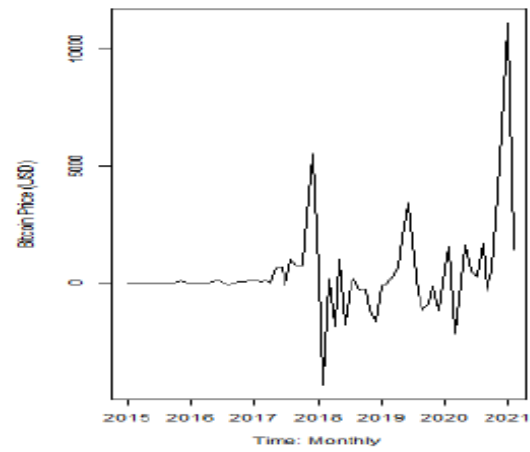


Figure M4: Monthly BTC Price (Difference)

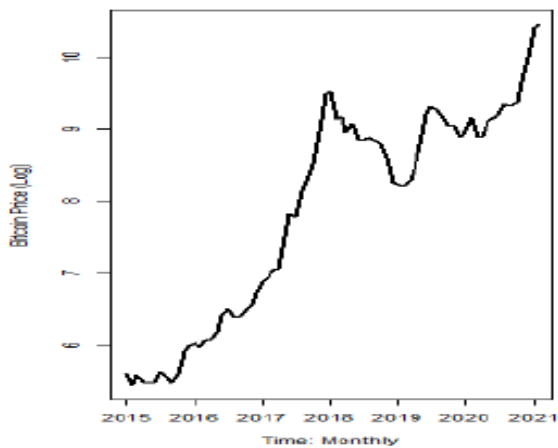


Figure M5: Log of Monthly BTC Price

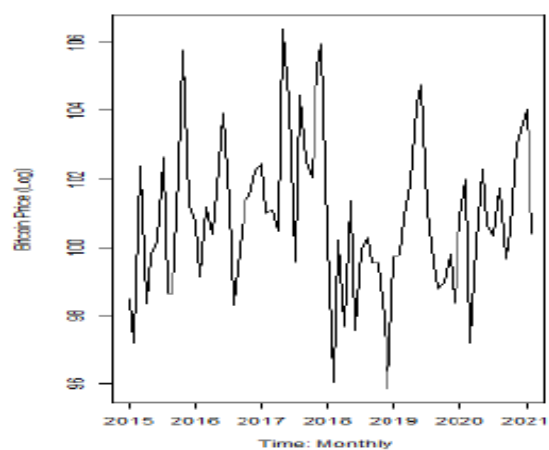


Figure M6: Log of Monthly BTC Price (Difference)

Date	PREDICTIONS					PREDICTION ERROR (%)									
	ACTUAL	NAIVE	LINEAR	ETS	ARIMA	NNAR	STL	HWM	NAIVE	LINEAR	ETS	ARIMA	NNAR	STL	HWM
01/07/19	11636.50	-	7836.60	11615.70	11307.10	11390.80	11590.90	11724.00	-2.29	-32.65	-0.18	-2.83	-2.11	-0.18	0.75
02/07/19	10697.60	11636.50	7841.70	11599.00	11321.70	11394.30	11631.30	11811.40	5.91	-26.70	8.43	5.83	6.51	8.43	10.41
03/07/19	10412.90	10697.60	7846.70	11585.70	11466.90	11429.30	11602.10	11898.90	9.01	-24.64	11.26	10.12	9.76	11.26	14.27
04/07/19	11201.50	10412.90	7851.80	11575.00	11507.30	11468.90	11578.70	11986.40	1.46	-29.90	3.33	2.73	2.39	3.33	7.01
05/07/19	11715.10	11201.50	7856.80	11566.50	11552.80	11518.10	11599.50	12073.90	-2.97	-32.93	-1.27	-1.39	-1.68	-1.27	3.06
06/07/19	11157.90	11715.10	7861.80	11559.60	11544.30	11571.60	11624.00	12161.40	1.87	-29.54	3.60	3.46	3.71	3.60	8.99
07/07/19	11193.40	11157.90	7866.90	11554.20	11622.80	11636.10	11612.10	12248.90	1.52	-29.73	3.21	3.82	3.85	3.21	9.41
08/07/19	11302.70	11193.40	7871.90	11549.80	11749.10	11680.80	11602.50	12336.40	0.95	-30.35	2.19	3.95	3.35	2.19	9.15
09/07/19	11728.60	11302.70	7876.90	11546.30	11697.90	11755.00	11638.00	12423.90	-2.75	-32.84	-1.55	-0.26	0.05	-1.55	5.93
10/07/19	12432.20	11728.60	7882.00	11543.50	11527.80	11787.80	11675.00	12511.40	-8.27	-36.60	-7.15	-7.27	-5.18	-7.15	0.64
11/07/19	12470.70	12432.20	7887.00	11541.30	11287.40	11838.60	11670.10	12598.90	-8.78	-36.76	-7.45	-9.49	-5.07	-7.45	1.03
12/07/19	11814.40	12470.70	7892.00	11539.50	11228.70	11887.90	11666.20	12686.40	-3.71	-33.20	-2.33	-4.96	0.62	-2.33	7.38
13/07/19	11479.60	11814.40	7897.10	11538.00	11164.60	11935.80	11717.90	12773.90	-0.69	-31.21	0.51	-2.74	3.97	0.51	11.27
14/07/19	11521.20	11479.60	7902.10	11536.90	11171.30	11982.80	11770.20	12861.40	-0.98	-31.41	0.14	-3.04	4.01	0.14	11.63
15/07/19	11022.30	11521.20	7907.10	11536.00	11229.50	12029.00	11768.20	12948.80	3.37	-28.26	4.66	1.88	9.13	4.66	17.48
16/07/19	10434.20	11022.30	7912.20	11535.30	11403.90	12075.00	11766.60	13036.30	9.14	-24.17	10.55	9.29	15.73	10.55	24.94
17/07/19	10447.10	10434.20	7917.20	11534.70	11855.60	12121.50	11767.60	13123.80	8.92	-24.22	10.41	13.48	16.03	10.41	25.62
18/07/19	9532.50	10447.10	7922.20	11534.20	12295.50	12168.60	11769.00	13211.30	19.57	-16.89	21.00	28.99	27.65	21.00	38.59
19/07/19	9937.40	9532.50	7927.20	11533.80	12308.00	12215.10	11768.10	13298.80	14.63	-20.23	16.06	23.86	22.92	16.06	33.83
20/07/19	10533.50	9937.40	7932.30	11533.50	12380.40	12260.70	11767.50	13386.30	8.12	-24.69	9.49	17.53	16.40	9.49	27.08
21/07/19	10675.30	10533.50	7937.30	11533.30	12269.50	12305.00	11838.00	13473.80	6.66	-25.65	8.04	14.93	15.27	8.04	26.22
22/07/19	10669.70	10675.30	7942.30	11533.10	12365.10	12348.30	11908.60	13561.30	6.87	-25.56	8.09	15.89	15.73	8.09	27.10
23/07/19	10467.90	10669.70	7947.30	11532.90	12450.40	12390.60	11908.30	13648.80	9.02	-24.08	10.17	18.94	18.37	10.17	30.39
24/07/19	10192.40	10467.90	7952.40	11532.80	12867.40	12432.00	11908.00	13736.30	11.59	-21.98	13.15	26.25	21.97	13.15	34.77
25/07/19	9803.60	10192.40	7957.40	11532.70	13212.90	12472.40	11872.90	13823.80	15.94	-18.83	17.64	34.78	27.22	17.64	41.01
26/07/19	9912.40	9803.60	7962.40	11532.60	13033.90	12511.60	11837.70	13911.30	14.69	-19.67	16.34	31.49	26.22	16.34	40.34
27/07/19	9849.20	9912.40	7967.40	11532.60	12936.20	12549.50	11837.60	13998.80	15.48	-19.11	17.09	31.34	27.42	17.09	42.13
28/07/19	9816.70	9849.20	7972.40	11532.50	13086.20	12586.20	11837.50	14086.20	16.11	-18.79	17.48	33.31	28.21	17.48	43.49
29/07/19	9439.80	9816.70	7977.50	11532.50	13125.00	12621.60	11781.10	14173.70	20.79	-15.49	22.17	39.04	33.71	22.17	50.15
30/07/19	9567.60	9439.80	7982.50	11532.40	13043.50	12655.80	11724.70	14261.20	19.08	-16.57	20.54	36.33	32.28	20.54	49.06
31/07/19	9553.80	9567.60	7987.50	11532.40	12938.50	12688.80	11724.60	14348.70	19.07	-16.39	20.71	35.43	32.81	20.71	50.19
01/08/19	9762.60	9553.80	7992.50	11532.40	12599.60	12720.70	11724.50	14436.20	16.64	-18.13	18.13	29.06	30.30	18.13	47.87
02/08/19	10148.80	9762.60	7997.50	11532.40	12542.10	12751.40	11682.20	14523.70	12.01	-21.20	13.63	23.58	25.64	13.63	43.11
03/08/19	10477.00	10148.80	8002.60	11532.40	12402.90	12781.00	11639.90	14611.20	8.74	-23.62	10.07	18.38	21.99	10.07	39.46
04/08/19	10656.50	10477.00	8007.60	11532.40	12225.40	12809.40	11639.90	14698.70	6.80	-24.86	8.22	14.72	20.20	8.22	37.93
05/08/19	10817.00	10656.50	8012.60	11532.40	11943.50	12836.70	11639.90	14786.20	5.19	-25.93	6.61	10.41	18.67	6.61	36.69
06/08/19	11272.20	10817.00	8017.60	11532.30	11958.60	12862.90	11651.20	14873.70	1.12	-28.87	2.31	6.09	14.11	2.31	31.95
07/08/19	11792.00	11272.20	8022.60	11532.30	11869.70	12888.10	11662.50	14961.20	-3.44	-31.97	-2.20	0.66	9.29	-2.20	26.88
08/08/19	11649.00	11792.00	8027.60	11532.30	11491.90	12912.10	11662.50	15048.70	-2.31	-31.09	-1.00	-1.35	10.84	-1.00	29.18
09/08/19	11829.90	11649.00	8032.60	11532.30	11312.10	12935.00	11662.50	15136.20	-3.50	-32.10	-2.52	-4.38	9.34	-2.52	27.95

Weekly predictions in the validation period

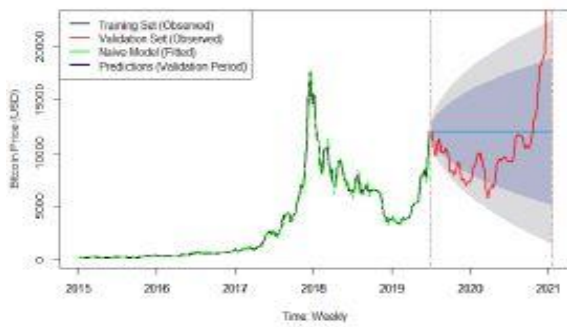


Figure 4a: Weekly predictions in the validation period (Naïve model)

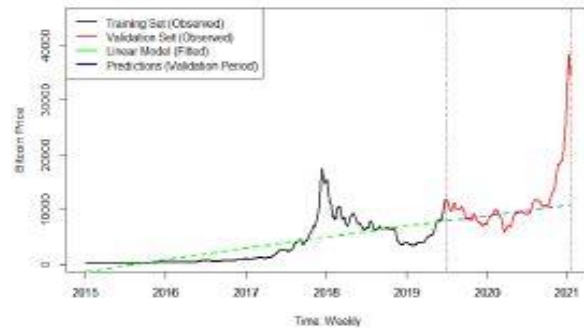


Figure 4b: Weekly predictions in the validation period (Linear model)

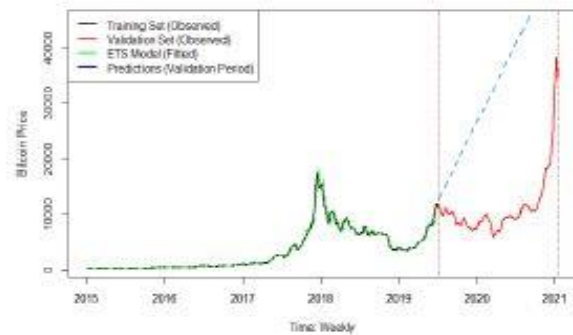


Figure 4c: Weekly predictions in the validation period (ETS model)

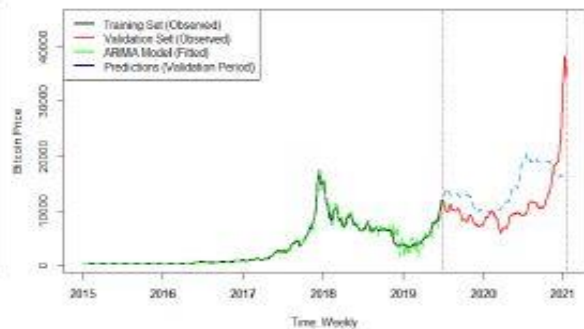


Figure 4d: Weekly predictions in the validation period (ARIMA model)

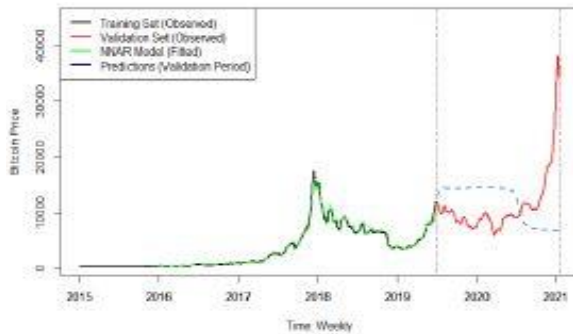


Figure 4e: Weekly predictions in the validation period (NNAR model)

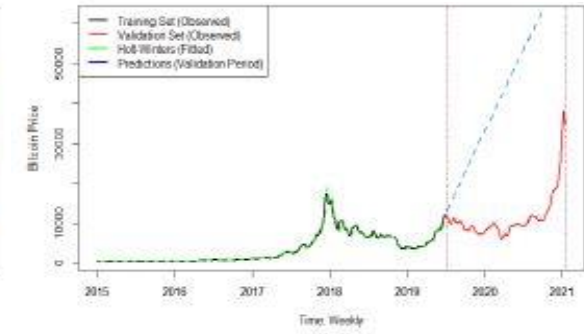


Figure 4f: Weekly predictions in the validation period (HWF model)

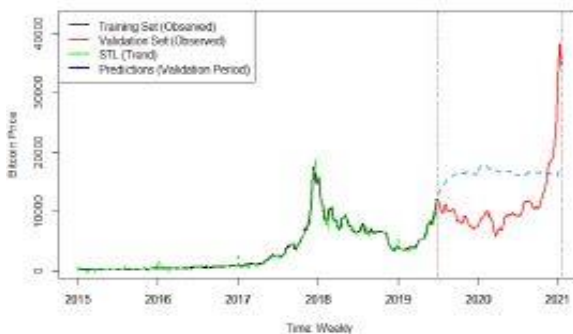


Figure 4g: Weekly predictions in the validation period (STL model)

Monthly predictions in the validation period

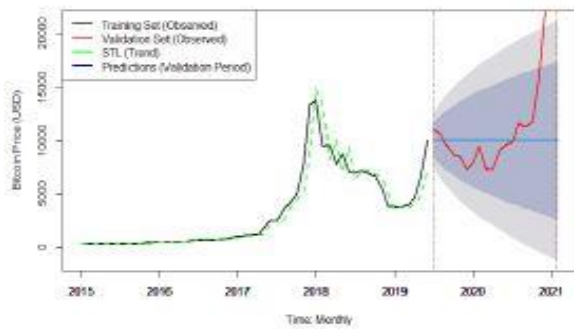


Figure 5a: Monthly predictions in the validation period (Naïve model)

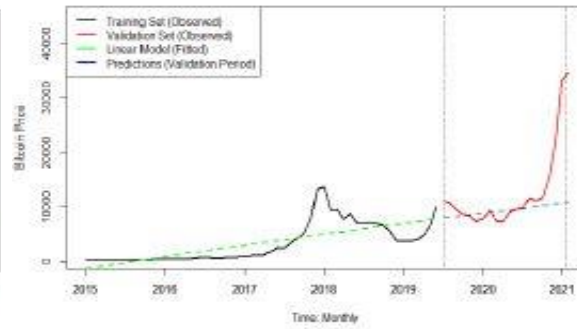


Figure 5b: Monthly predictions in the validation period (Linear model)

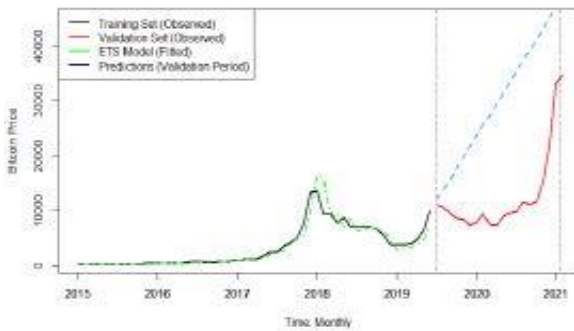


Figure 5c: Monthly predictions in the validation period (ETS model)

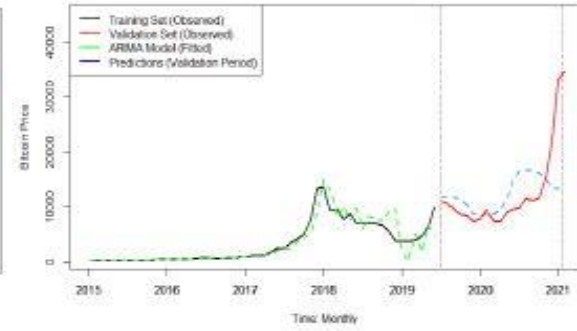


Figure 5d: Monthly predictions in the validation period (ARIMA model)

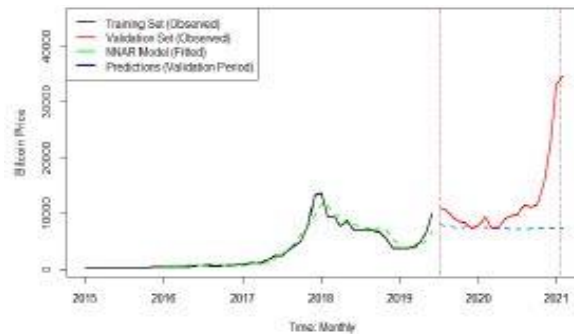


Figure 5e: Monthly predictions in the validation period (NNAR model)

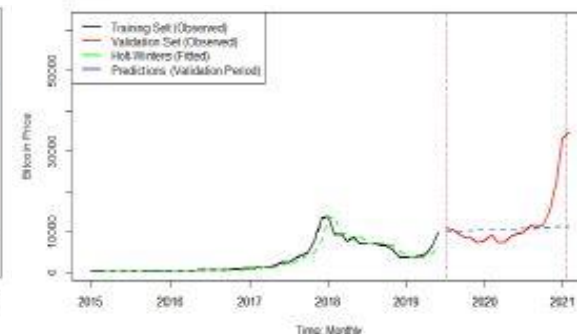


Figure 5f: Monthly predictions in the validation period (HWF model)

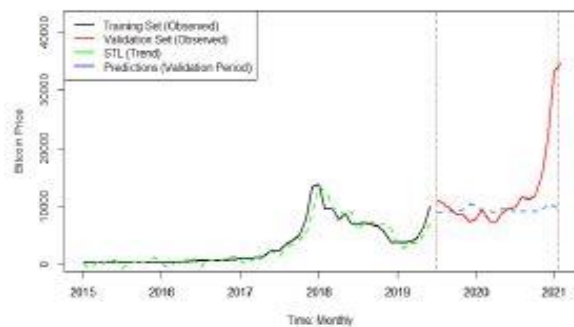


Figure 5g: Monthly predictions in the validation period (STL model)

Table D.2: Autocorrelation and conditional heteroscedasticity tests

Forecast Methods	Test	Daily		Weekly		Monthly	
		Stat.	p-value	Stat.	p-value	Stat.	p-value
NAIVE	LBox (\tilde{Q}^*)	885.20	(2.20×10^{-16})	101.71	(6.06×10^{-6})	8.2993	$(0.686343)^{***}$
ETS	LBox (\tilde{Q}^*)	552.19	(3.02×10^{-14})	54.494	$(0.1124)^{***}$	4.9994	$(0.623120)^{***}$
LINEAR	B-G (LM)	1634.3	(2.20×10^{-16})	228.13	(2.20×10^{-16})	44.965	(4.920×10^{-6})
ARIMA	LBox (\tilde{Q}^*)	627.96	(2.20×10^{-16})	115.35	(6.99×10^{-8})	3.8399	$(0.9543)^{***}$
NNAR	LBox (\tilde{Q}^*)	513.31	(6.96×10^{-11})	63.276	(0.0185)	3.0353	(0.980623)
STL	B-G (LM)	1534.3	(2.30×10^{-15})	-	-	4.2532	(0.882145)
HWM	LBox(\tilde{Q}^*)	563.27	(3.20×10^{-16})	87.255	(3.470×10^{-5})	51.483	(1.33×10^{-5})
NAIVE	$\tilde{Q}M(x^2)$	0.6483	$(0.9996)^*$	1.7075	(0.9887)	1.6734	(0.991271)
ETS	$\tilde{Q}M(x^2)$	15.865	(0.0072)	39.657	(1.75×10^{-7})	17.745	(0.003284)
LINEAR	$\tilde{Q}M(x^2)$	10.883	(0.0009707)	10.234	(39.657)	7.9473	(0.004816)
ARIMA	$\tilde{Q}M(x^2)$	9.9624	(0.0016)	9.7093	(0.0018)	8.2571	(0.004059)
NNAR	$\tilde{Q}M(x^2)$	15.674	$(7.53 \times 10^{-5}^*)$	15.161	(9.88×10^{-5})	9.7093	(0.001833)
STL	$\tilde{Q}M(x^2)$	9.5214	(0.0023)	6.5773	(0.0103)	1.1091	(0.292334)
HWM	$\tilde{Q}M(x^2)$	9.8933	(0.0017)	9.2731	(0.0023)	7.2250	(0.007190)

Breusch-Godfrey [B-G (LM)]: Ljung-Box (\tilde{Q}^*) (x-square); $\tilde{Q}M(x^2)$ (p-values < 0.05) indicates the presence of conditional heteroscedasticity.

*** indicate no autocorrelation

