International Journal of Business and Economic Sciences Applied Research

IJBESAR
ijbesar.ihu.gr

Price Prediction for Bitcoin: Does Periodicity Matter?

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ARTICLE INFO

Article History

Received 18 June 2022
Accepted 1 November 2022

JEL Classifications
C10; G15, G17

ABSTRACT

Purpose:
A major challenge traders, speculators and investors are grappling with is how to accurately forecast Bitcoin price in the cryptocurrency market. This study is aimed to uncover the best model for the forecasts of Bitcoin price as well as to verify the price series that offers the best predictions performance under different periodicity of datasets.

Design/methodology/approach:
The study adopts three different data periods to verify whether frequency matters in forecasting Bitcoin price. The Bitcoin price, from 01/01/15 to 11/01/2021, is trained and validated on selected forecast models, including the Naïve, Linear, Exponential Smoothing Model, ARIMA, Neural Network, STL and Holt-Winters filters. Five forecast accuracy measures (RSME, MAE, MPE, MAPE and MASE) are applied to confirm the best performing model. The Diebold-Mariano test is used to compare the forecasts based on the daily price with those based on the weekly and monthly.

Findings:
Based on the accuracy measures, the results indicate that the Naïve model provides more accurate performance for the daily series, while the linear model outperforms others for the weekly and monthly series. Using the Diebold-Mariano statistics, there is evidence that forecasting Bitcoin price is not sensitive to the data periodicity.

Research limitations/implications:
The study has a major limitation, which is the shared sentiment to apply actual Bitcoin price series, and not the returns or log transformation for the forecast models. Notably, actual data may sometimes be loud, hence increasing the possibility of over predictions.

Originality/value:
In forecasting, different approaches have been used, this paper compares outputs of both statistical and machine learning methods in order to arrive at the best option for the Bitcoin price forecasts. Hence, we investigate whether the machine learning tools offer better forecasts in terms of lower error and higher model's accuracy relative to the traditional models.

Keywords:
Bitcoin, Diebold-Mariano test, Univariate forecast models, Forecast accuracy

1. Introduction

There is increasing research on Bitcoin (BTC) in the fields of theoretical and empirical finance. Bitcoin is a cryptocurrency that relies on anonymous peer-to-peer trades via online and social networks interfaces. Its transactions are organised on the Blockchain, an open-source algorithm that uses sophisticated protocol to generate and verify records. Bitcoin shares known attributes with typical financial assets (Baur et al., 2018; Mikhaylov, 2020), and has been exploited as medium of payments as well as accepted in exchange for alternative cryptocurrencies and different national currencies. Bitcoin stands as a speculative asset in times of economic upheavals (Baur et al., 2018), and sometimes perceived as a safe haven and substitute for traditional financial assets (Kliber et al., 2019). During the wave of COVID-19, the price of Bitcoin soared higher relative to conventional assets and commodities (Hung et al., 2020). Bitcoin remains unregulated by any coordinated monetary policy of central banks (Barontini & Holden, 2020).

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DOI: 10.25103/ijbesar.153.06

However, there are reports on the plan to create Central Bank Digital Currency to regulate Bitcoin and Digital Ledgers (Bofinger & Haas, 2021; IMF, 2020; Auer et al., 2020).

The price of Bitcoin is associated with consistent short- and long- term volatility. The fluctuations in the price is mostly attributed to the limited supply, demand increase, activities of trend chasers and speculations in the bitcoin market. The excessive swings have immersed pressure on users, investors and regulators, leading to increasing interests to forecast its price (Aalborg et al., 2018; Kliber et al., 2019). Studies that focus on forecasting the price of bitcoin use either intraday, daily, weekly or/and monthly series (Bouri et al., 2021; Sitzimis, 2021; Uras et al., 2020). Bouri et al. (2021) employ the functional forecasting approach to examine the intraday trading under the efficient market hypothesis. They provide evidence of profitable trades based on the trading strategies. The bitcoin cumulative intraday return is observed to be heteroscedastic, stationary, non-normal and uncorrelated. Uras et al. (2020) forecast the daily price of bitcoin using different statistical techniques. The authors note that the price appears to be indistinguishable from a random walk process. When the dataset is partitioned into shorter sequences, the evidence confirms the regime hypothesis.

Forecasting the price of Bitcoin has implications for the financial markets. Suitable forecast models offer traders the realistic direction of price, including information on whether to transact on the spot or future markets. The models serve as tools that help investors to circumvent massive losses from sporadic volatility. An accurate forecast model provides the opportunity to increase returns and trading (Munim et al., 2019), since the asset managers would avoid risk by employing the model with least possible error (Kliber et al., 2019). The choice of a forecast model is challenging due to asymmetric information, uncertainties and dynamic behaviours of miners. This study intends to find the best forecast model for Bitcoin price, and on the basis of the different periodicity of datasets, verifies the series that offers the best forecast performance.

We contribute to existing literature in two ways. First, we compare outputs of statistical and machine learning methods in order to arrive at the best option for the Bitcoin price forecast. Forecasting with these approaches have been used in different fields of research (Basher & Sadorsky, 2022; Ye et al., 2022; Chen et al., 2020; Rizwan et al., 2019), including specific application to passenger traffic in coastal shipping (Sitzimis, 2021). We examine whether the machine learning tools offer better forecasts than the traditional models, in terms of lower error and higher accuracy of the model. This becomes necessary in the light of the continuous applications of machine learning approaches which outputs often depict distinct forecast patterns. We train and validate the Bitcoin price series on selected forecasting models as well as compute alternative forecast accuracy to decide the best suitable model. Second, we consider the issue of data frequencies using daily, weekly and monthly series. We check whether the forecast models of Bitcoin price are sensitive to data frequency. The need to test the resilience of periodicity becomes important as the result would offer lead on best choice of dataset to evaluate bitcoin price forecasts, and by extension other alternative cryptocurrencies.

The result shows that for the daily time-series the Naïve model outperforms the others. The evidence based on the Diebold-Mariano statistics indicates that forecasting the Bitcoin price is not sensitive to the data frequency. The rest of the paper is organised as follows. Section two presents a brief trend movement of Bitcoin price. Section three is the material and methodology, where the study summarises the various forecast models and present some measures of forecast accuracy. Section four presents the results including the summary statistics, stationarity tests, forecast models, and the forecast accuracy. Section five is the conclusions.

2. Materials

2.1 Bitcoin Price Trends

Although Bitcoin was reportedly invented in 2009, it first featured on a cryptocurrency exchange on February 6, 2010. Since then, it has witnessed unprecedented and continuous price movements. On March 18, 2013, the US Financial Crimes Enforcement Network issued regulations on virtual currency and legal recognition of bitcoin, and this was believed to motivate the significant increase in bitcoin price from USD149.08 on October 15 to about USD1,242 on November 29. In 2014, there was massive price decline caused by the hacking of the then biggest Bitcoin exchange (Mt. Gox), making the price to rally around USD340.00–USD531.05. The price decline continued and stood at USD434.25 at 2015 end. The Bitcoin splits (hard forks) on August 1, 2017, marks monumental strides in BTC price rallies, with massive run up (buy orders), pressuring the price to reach an all-time high of USD19,783.06 on December 17, 2017.

The increase could not be sustained, therefore the price dropped to USD13,412.44 by January 1, 2018. Figure 1 shows the daily price from July 1, 2018, to June 30, 2021. The price experience massive run-up, resistance, reversals, different supports and consolidations. The price dropped to USD6,300 on October 31, 2018, and dipped further below USD3,300 by December 7, 2018. The price started above USD3,700 in 2019, and stood at USD7,200 by year end. In November 2020, the price rallied above USD18,000, regaining its losses from 2017 peak. The price later surpassed its previous peaks, crossed above USD40,000 and landed on a remarkable daily average all-time high of about USD 64,863.31 on April 14, 2021. The price has fallen about 40% to USD40,044.54 in June 2021.

DOI: 10.25103/ijbesar.153.06

Figure 1: Daily Price of BTC in USD
Source: (Author's construct, 2023)

2.2 Empirical Highlights

Time-series literature recommends model-based and univariate-based methods for forecasting volatile assets. The first approach predicts bitcoin price as dependent on some factors (Gbadebo et al., 2021; Jaquart et al., 2021; Koutmos & Payne, 2020; Liang et al., 2020). Gbadebo et al. (2021) employ the Autoregressive Distributed Lag (ARDL) to verify how Bitcoin price volatility responds to cryptocurrency capitalisation, equity index, trading volume and Google search. The study confirms the existence of long run cointegration and conclude that market fundamentals drive the volatility of price than information demand. Jaquart et al. (2021) use artificial neural network (ANN), random forests (RF) and long short-term memory (LSTM) to analyse how blockchain, technical, sentiment and asset returns explain Bitcoin price forecast. The quantile result shows the long-short trading strategy creates about 39% returns. Liang et al. (2020) apply the GARCH-MIDAS model to investigate competing index predictors. They provide that the Chicago Board Options Exchange (CBOE)'s gold volatility index exhibits strongest predictability for the BTC price volatility relative to the CBOE volatility index, google trends, global economic policy uncertainty and geopolitical risk. Koutmos (2020) uses a Markov regime-switching model to show that asset pricing factors such as stock price, interest rate and exchange rates are the main determinants of Bitcoin price.

The application of the model-based approach has notable limitations, including depending on prior assumptions made about the series' distribution. As noted, (Aalborg et al., 2018), predicting Bitcoin price on the basis of these fundamental indicators is still ambiguous. Hence, the second approach based on univariate times-series would be more suitable for forecasting the Bitcoin price. Caporale et al. (2018) establish the existence of correlation amongst past and present values of the BTC price. Many studies (Basher & Sadorsky, 2022; Ye et al., 2022; Aygün & Günay Kabakçı, 2021; Chen et al., 2020; Munim et al., 2019; Adcock & Gradojevic, 2019; Mallqui & Fernandes, 2019; Rizwan et al., 2019; McNally et al., 2018) confirm the robustness of the univariate approach. Ye et al. (2022) apply an ensemble machine learning model to forecast Bitcoin's next prices. They combine both the LSTM and Gated Recurrent Unit (GRU) with stacking ensemble system and use sentiment indexes, technical indicators to forecast Bitcoin prices, during September 2017 to January 2021. The results indicate that the near-real time forecast exhibit better performance MAE of 88.74%. Basher and Sadorsky (2022) use random forests and bagging classifiers and the logit models to predict Bitcoin prices. The accuracy for the random forests and the bagging classifiers range above 85% for 10 to 20 days prediction and between 75% and 80% for the 5-day forecasts. They conclude that the random forests predict the Bitcoin price with much accuracy than the logit models. Aygün and Günay Kabakçı (2021) explore the MA, ARIMA as well as machine learnings (ANN, RNN) and convolutional neural network (CNN) of Bitcoin price predictions. The RNN offers better performance relative to other methods. Hamayel and Owda (2021) employ three machine learning methods (LSTM, bi-LSTM and GRU to predict Bitcoin, Litecoin, and Ethereum. The GRU model show the smallest MAPE and RMSE, outperforming other algorithms.

Chen et al. (2020) compare support vector machine (SVM) and long short-term memory (LSTM) and showed that, for the next day BTC price, the SVM provides a higher accuracy of 65.3% classification. Demir et al. (2019) predict the price of Bitcoin using methods such as long LSTM, NB, as well as the nearest neighbour technique. These methods achieved prediction accuracy between 81.2% and 97.2%. Mallqui and Fernandes (2019) employ artificial neural network (ANN) and support vector machines (SVM) algorithms in regression models to forecast the maximum, minimum and closing Bitcoin prices. He concludes that SVM algorithm outperformed the ANN with lowest mean absolute percentage error (MAPE) of 1.58%. McNally et al. (2018) employ the Bayesian recurrent neural network (RNN) and LSTM to forecast the daily movement in the price of Bitcoin. The LSTM achieve a high performance with the classification accuracy of 52% and a root mean squared error (RMSE) of 8%. Munim et al.

DOI: 10.25103/ijbesar.153.06

(2019) employ an autoregressive integrated moving average (ARIMA) and a neural network autoregression (NNAR). They split the data into two training-sets, and for the first training-set, the NNAR outperforms the ARIMA, while for the second, the ARIMA outperforms the NNAR. Velankar et al. (2018) use the generalized linear (GLM) model and Bayesian regression to forecast the daily average price change signals and uncover a prediction accuracy rates of 51% with the GLM. Adcock and Gradojevic (2019) use the feed-forward neural networks (FNN), GARCH-M, ARIMAX, random walk and multiple regression to predict prices. They examine how 50-200 days moving averages (MA) of bitcoin volume and VIX affect its prices, which shows little significance on its forecasts. The FNN indicates the highest accurate density and point forecast relative to other models.

3. Methodology

3.1 Forecast models and predictive accuracy

Organizational the study employs univariate-based forecast models. Each model is evaluated based on the accuracy of its predictions *vis-à-vis* actual data. We adopt five methods (RMSE, MAE, MPE, MAPE and MASE) to assess the accuracy of the forecast methods. To avoid the over-fitting problem, we trim the time-series into two sets: Training and validation (test) sets. We scrutinise the data behaviour as well as consider the data frequency and forecast horizon in deciding the length for the validation periods (Hyndman & Athanasopoulos, 2021). We select a forecast horizon which does not exceed the validation periods to arrive at training-set (01\01\15–30\06\19) and validation-set (01\07\19–11\01\2021) for the daily time series. The weekly has training (01\01\15 – 27\06\19) and validation (28\06\19 – 11\01\2021), while the monthly is trained on (01\01\15 – 01\07\19) and validated on (01\08\19 – 11\01\2021). The forecast errors of the models in Table 1a are used to compute the accuracy measures. Table 1b presents the various measures of forecast accuracy.

3.2. The Data

We employ Bitcoin price from the Finance.yahoo's official website. The database stores historical data on Bitcoin price from the real time price on the CoinMarketCap Exchange. The daily data obtained, spanning 01\01\15 to 11\01\21, reports the opening, lowest, highest and closing prices. We apply the closing price in line with previous studies (Uras et al., 2020; Chen et al., 2020; Munim et al., 2019). Previous studies apply daily data (Uras et al., 2020; Chen et al., 2020), while some others employ weekly (Othman et al., 2020) and/or monthly (Ramadhani et al., 2018) series for forecasting bitcoin price. Because we aim to verify whether periodicity matters in the performance of the forecast, we use three different datasets.

In this paper, we do not apply log transformation for the different series used. We share the sentiments to verify the Bitcoin price forecasts in its original form because there are downside to forecasting security prices or returns in logarithm (Hudson & Gregoriou, 2010) or other transformation forms (Meucci & Quant, 2010). As noted, (Hudson & Gregoriou, 2010), the mean of a set of random variables computed using logarithmic is often less than the mean computed from the simple set, specifically by an amount dependent on the variance of the set. In effects, when the log series are applied, ceteris paribus, higher variance will inevitably reduce the mean price or returns.

Table 1a: Summary forecast models

Model	Explanation	Model Algorithms (Equations)	References
Naïve Model (NAÏVE)	Naïve model uses observations of the previous period to forecast the next. The method takes the last observation as the forecast. Let y_t $(t = 1, 2 \cdots, T)$ denotes Bitcoin price, y_T as actual value of the last observation. Divide y_t to: training set (t = 1, 2,...,n) and validation set [$t = n + 1, n + 2, \ldots, n + v$ (=T)]; e_t is forecast error.	$\hat{y}_{T+h\|T} = y_T$ $e_t = y_T - \hat{y}_{T+h\|T}$ $(\hat{y}_{T+h\|T})$ = h-step forecast.	Stenqvist & Lonno (2017)
Linear Trend (LINEAR)	Linear trend creates forecasts values been a generalisation of y_t as a time-trends. The trend-line approach is used if the y_t series exhibits steady increase or decrease overtime and an error term (ε_t). A polynomial function for y_t depends on Trend (T), trend square (T^2) and a drift (ψ_0).	$y_t = \psi_0 + \psi_1 T + \psi_1 T^2 + \varepsilon_t$; $\hat{y}_{T+h\|T} = y_t + e_t$; $\varepsilon_t = \hat{y}_{T+h\|T} - y_t$	Bisht & Agarwa (2017) Ostertagová (2012)
Exponential	The ETS creates forecast weighted	$y_t = \mu_t + \beta_t t + S_{t,p} + \varepsilon_t$	Liantoni &

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Smooth Model (ETS)	averages with the recent observations more weighted than distant ones when determining the forecasts. The weights ($\emptyset_j$) diminish exponentially [$\emptyset_j = \emptyset^j$;$-1 < \emptyset^j < 1$ $(j = 1,2,\cdots m)$]. The 3 (time-varying) components: mean (μ_t), slope (β_t) and seasonality, $S_{t,p}$ $(p = 1,2,\cdots,P)$ for p seasons, $\forall$ t, $\beta_t = 0$ & $\forall\, p$, $S_{t,p} = 0$. The smoothing starts by computing at $t = 1$. T_t is smoothed slope that estimates β_t, L_t is smoothed level that estimates μ_t; S_t is smoothed seasonality that estimates $S_{t,p}$. Initial estimates smoothing-states: $t = 0$: L_0, T_0, and $S_{0,1},\cdots,S_{0,P}$ use for the Smoothing equations (L_t, S_t *and* T_t).	$L_t = \alpha(y_t - S_{t-P}) + (1-\alpha)L_{t-1}$ $S_t = \delta(y_t - L_t) + (1-\delta)S_{t-P}$ $\hat{y}_{T+h\|T} = L_t + S_{t-P+h}$ Invertible region: $\max(-P\alpha, O) < \delta(1-\alpha) < 2 - \alpha$.	Agusti (2020) Olvera-Juarez & Huerta-Manzanilla (2019)
ARIMA Model	ARIMA has an autoregressive [AR(p)], a moving average [MA(q)] and an order of integration components, where d is the number (#) of differencing required to attain a stationary [ARMA (p, q)] model and q is the order of the MA component. μ is the intercept (drift time-series, which is often zero), y_{t-i} $(i = 1, \ldots p)$ is previous time series periods until lag p, θ_i is the parameter for y_{t-i}, ε_t is the error term in time t, ε_{t-j} is the error term of all previous periods until lag q and δ_j $(j = 1, \ldots q)$ is the parameter for ε_{t-j}.	General ARIMA (p, d, q) model is: $\Delta y_t = \mu + \sum_{i=1}^{p} \theta_i\, \Delta y_{t-i} + \sum_{j=1}^{q} \delta_j\, \varepsilon_{t-j} + \varepsilon_t$ $\Delta y_t = y_t - y_{t-i}$	Munim *et al.* (2019) McNally *et al.* (2018) Bakar & Rosbi (2017)
NNAR Model	NNAR is a sophisticated neurone-like elements assembled in layers. While simple NNAR is analogues to linear regression model with inputs (predictors) and output (dependent variable), the complex NNAR is nonlinear. y_j is actual state of output unit j in the input-output; X_j is the input $-$ vector; α_j is constant for node j, $W_{i,j}$ is weight-vector from input node i to output node j, and m is # of inputs. The parameters $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ & $W_{1,1}$, $\ldots W_{4,3}$, are 'learned' from training data. Before training, we restrict $W_{i,j}$ & set as 0.1. If y_j is transformed via sigmoid squashing, we get $s(y)$, where β, k, c and τ are constants. The learning is reduced to a minimum error with repeated changing of $W_{i,j}$ by an amount (δ_j) proportional o $\partial E/\partial W_{i,j}$. $\hat{y}_j$ is	NNAR error back-propagation algorithms: $y_j = \alpha_j + \sum_i^m W_{ij}X_j$ $s(y) = \beta + \frac{k}{1+e^{\tau y}}$ $E = \frac{1}{2}\sum_J^n(y_J - \hat{y}_j)$ $\Delta W_j(t+1) = \lambda\delta_j y_j$ $\delta_j = (\hat{y}_j - y_j)f'(y_j)$ $(y_{t-1}, \ldots, y_{t-p}, y_{t-m}, \ldots\, y_{t-Pm})$	Chen *et al.* (2020) Munim *et al.* (2019) Mallqui & Fernandes (2019) McNally *et al.* (2018)

DOI: 10.25103/ijbesar.153.06

	desired state and the learning rate, λ is kept constant. To forecast with the NNAR, the lagged values of the univariate series is used as inputs. A feed-forward NNAR with one hidden layer is denoted NNAR(p, k) or $\text{NNAR}(p, P, k)_m$, where p is lag-length or p last observations used as inputs, k is the # of nodes (neurons) in the hidden layer and p is # of seasonality.		
STL Model	STL adopts a non-parametric algorithm that iterates loess smoother to refine y_t into 3 components. y_t consists of a trend (T_t), a seasonality (S_t) and an irregularity (l_t). The STL assumes S_t has the same cycle periodically. The cycle adopts a spectral analysis which shows the characteristics of oscillations of different wavelengths. The spectrum of a process y_t with an autocorrelation function (ω_τ) where, $\sum_\tau^n \mid \omega_\tau \mid < \infty$ is denoted $y(\omega_\tau)$. STL protocol set for T_t smoothing parameter is: *t.window* $\geq [\frac{1.5*frequency}{1-\frac{15}{s.window}}]$ (must be odd integer $\geq$ 7).	$y_t = T_t + S_t + l_t$ (additive split) $y_t = T_t * S_t * l_t$ (multiplicative split) $y(\omega_\tau) = \omega + 2\sum_{\tau=1}^{\infty} \omega_\tau \cos(2\pi\omega\tau)$	Hyndman & Athanasopoulos (2021)
Holt-Winters Model (HWM)	HWM is a typical deterministic model with a trend, seasonality and residuals. HWM computes a smooth series $\hat{y}_{T+h\|t}$ with recursively updating equations that allow for the iterative computation of forecasts based additive or multiplicative protocols. The additive algorithm is criticised not to generate best estimates for time-series level and seasonality. We adopted a multiplicative algorithm, which assumes the seasonal effect is proportional to a time change. The level (p_t), trend(b_t), and seasonality (s_t) which depend on the smoothing parameters α , β, $\gamma \in [0, 1]$. The forecast *h*-step at time $T + h$ given data up to time t, and the constant k is the seasonality. The estimation of α , β, and γ is through the minimisation of randomly chosen errors. To estimate $\alpha, \beta, \gamma \in [0, 1]$, a robust smoothing process centred on *M*-estimation uses: $\hat{y}_{T+h\|t} = \text{arg min} \sum_{t=1}^{T} {e_t}^2$. We presented forecast for HWM with trend but no seasonal component HWM(γ[False]).	$y_t = \mu_t + \beta_t t + S_{t,p} + \varepsilon_t$ $p_t = \alpha \frac{y_t}{s_{t-k}} + (1-\alpha)(p_{t-1} + b_{t-1})$ $b_t = \beta^*(p_t - p_{t-1}) + (1-\beta^*)b_{t-1}$ $s_t = \gamma \frac{y_t}{p_{t-1} + b_{t-1}} + (1-\gamma)s_{t-k}$ $\hat{y}_{T+h\|t} = (p_t + hb_t)s_{t-k+h_k^+}$	Brügner (2017) Kuang *et al.* (2016)

Table 1b: Predictive accuracy measures

DOI: 10.25103/ijbesar.153.06

Accuracy measure	Accuracy scale/computation
RMSE	$\left(\frac{1}{m}\sum_{t=1}^{m}(e_t)^2\right)^{1/2}$
MAE	$\frac{1}{m}\sum_{t=1}^{m}\lvert e_t\rvert$
MPE	$\frac{100}{m}\sum_{t=1}^{m}\frac{e_t}{y_t}$
MAPE	$\frac{100}{m}\sum_{t=1}^{m}\lvert\frac{e_t}{y_t}\rvert$
MASE	$\frac{1}{m}\sum_{t=n+1}^{n+m}\lvert e_t\rvert \Big/ \frac{1}{n-1}\sum_{t}^{n}\lvert y_{t-1}-y_t\rvert$

Mean Absolute Error (MAE) gives the magnitude of the average absolute error in all periods. Root Mean Square Error (RMSE) and Mean Absolute Percent Error (MAPE) provide a percentage score of how forecasts deviate from the actual values. MASE compares predictive model performance to the Naïve forecast on the training set.
Source: (Author's construct, 2023)

3.3. Estimation Process

We adopt a static forecast approach for estimation. The approach ensures the univariate variable's actual value in previous periods is employed to estimate each step forecast. We follow the standard process of time-series forecasting, identifying the time series into training sample (observed datasets) and validation samples (observed datasets). We model the data with training samples and evaluated the forecast performance with validation samples. We combine the series, train the model on the full observed data and use the performance to forecast future prices. The data are trained on all forecast models with training datasets. We select a test period that mimics the predictive horizon for the future forecasts' valuation of performance.

We adopt library (*forecast*) and library (*fpp2*) in RStudio. We apply the stl (time series, *s.window* ="periodic") function to decompose y_t by obtaining T_t using loess and calculate S_t (and I_t) as $y_t - T_t$. By default setting for the *s.window* parameter, the function stl() assumes S_t follows the same cycle yearly. To ensure equal-spaced data, the study resolves the problem of non-multiple integer periodicity in infra-monthly high-frequency data by following Hyndman and Athanasopoulos (2021). The study periods have two leap-year-days (29\02\2016 and 29\02\2020). We set the frequency at 365.25 for daily series with the function ts(dataset, start = c(2015, 1), frequency = 365.25).

In the computation of the HWM, the study omits the seasonal component then set the function *Holt-Winters* (dataset, gamma = false) which allows for 365 - long vector of the initial seasonal pattern as its argument. We could not do otherwise since the Holt-Winters function (dataset, gamma = "integer") requires frequency to be multiple of the length of observations for the forecast to be computed in the next cycle. The ets() functions ignore the seasonality for infra monthly data with a frequency greater than 24 during computation. The function *auto.arima*() library in R selects and returns best ARIMA model through AIC, AICc or BIC[1] values. The order of the ARIMA model was selected through automatic iteration. The nnetar() function fits an $\mathrm{NNAR}(p, P, k)_m$ model. If the values of p and P are not defined, the lag is selected automatically according to the AIC for a linear $\mathrm{AR}(p)$ model.

Before we proceed to forecasting, we complete three diagnostic tests - Box-Ljung (BL) autocorrelation test, Box-Pierce (BP) x-square residual test and the Jarque-Bera (JB) normality test to determine the validity of the forecast models. The LB test is a portmanteau test for the "overall" randomness based on some lags, with the test null that the residuals from the forecast model (fitted) have no autocorrelation. The BP test with a test statistic (*Qm*) verifies whether the series is pure white noise. The Diebold-Mariano (DM) test compares two forecast models. It determines whether one forecast model is more accurate than the other.

4. The Results

4.1. Data statistics

Table 2 presents the deterministic statistical properties for the price of Bitcoin for each periodicity, including their training-set and validation-set partitions. The table shows that all series are asymmetrically distributed with positive skewness. For the training and validation sets, the daily dataset with 1.299 and 3.038 degree of skewness, respectively appears more skewed compared to other frequencies. The training samples appear to be mesokurtic (moderately peaked), while the others are leptokurtic (high peaked) for all frequencies. The outliers are more on the validation

[1] Autocorrelation Function (ACF); Partial Autocorrelation Function (PACF); Akaike Information Criterion (AIC); corrected Akaike Information Criterion (AICc); Bayesian information criterion (BIC).

DOI: 10.25103/ijbesar.153.06

samples. We reject the normality null for all the data partitions with a highly significant Jarque-Bera test. The Bitcoin price plot (Figure 1a) supposes the data may not be stationary. The non-stationarity would be confirmed with the unit root test.

Figure 1a –1f represent plots for the daily Bitcoin price (full data), the training sets, validation periods, the first difference, log daily price and the log-difference. The weekly (Figure W1 – W6) and monthly (Figure M1 – M6) plots are presented in the appendix. The plots replicate same shape with the daily plots, except that the infra-monthly plots show more volatility, outliers, and breaks. The daily series presents multiple, non-integer periodicities associated with high volatility with microstructure effect (Urquhart, 2018), while the monthly series appear smoother with less clustering. All observed series are chaotic with spiky protrusions. The log-transformed series appear with smoother striations.

4.2. Time-series decomposition

Figure 2a – 2c present the decomposition of daily, weekly and monthly. We apply the stl() function to decompose the observed data (topmost graph) into key time-series components. The function segregates the deterministic ('trend' and 'seasonal') and stochastic ('random') components of the Bitcoin price series. We apply the daily, weekly and monthly seasonal window. The trend component reflects the long-term progression (upward movement) of the series over-time, while the remainder (residual) is convergence with mean reversing. The seasonality is oscillatory with repetitive pattern over-time. In the daily series, the trend appeared unchanged and stable around January 2015 to February 2017. After these periods, the frequency and amplitude of the cycle upsurge over time. With the Loess framework, Bitcoin price shows exponential trends upward with additive seasonality. The residuals are quite random, particularly exhibiting high variability around late 2017 during the first remarkable price peak. Table 3 presents the summary statistics of the STL decomposition.

Table 2: Data deterministic statistics

	Daily			Weekly			Monthly		
Statistics	Training	Validation	Full	Training	Validation	Full	Training	Validation	Full
Mean	3365.8	10923.4	5290.4	3368.0	11299.3	5401.1	3346.8	12797.2	5901.0
Median	1184.6	9641.5	4141.9	1166.0	9607.2	4255.5	1140.8	9696.3	4411.3
Maximum	19513.0	40402.0	40402.0	17517.1	38255.1	38255.1	13742.3	34662.5	34662.5
Minimum	194.3	4987.6	194.3	194.3	5791.6	194.3	231.5	7285.0	231.5
Std. Dev.	3726.3	4978.3	5243.7	3713.6	5779.0	5545.8	3667.2	7983.7	6649.0
Skewness	1.3	3.0	1.6	1.2	2.9	1.9	1.1	2.0	2.2
Kurtosis	4.4	14.2	8.3	4.1	12.0	9.8	3.3	5.7	9.6
JB(Stat)	595.8	3787.8	3568.8	73.4	384.9	795.1	11.0	19.2	195.9
JB (p-value)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0041	0.0001	0.0000
# of Obs.	1642	561	2203	235	81	316	54	20	74

Note: JB: Jarque-Bera, # of Obs.: Number of Observations

Source: (Author's construct, 2023)

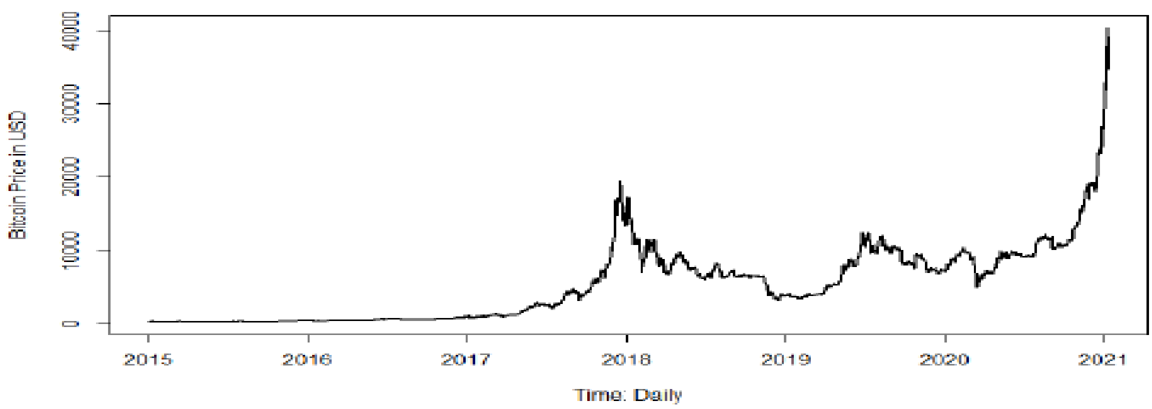

Figure 1a: Daily Bitcoin Price in USD (01-15-21 to 11-01-21)

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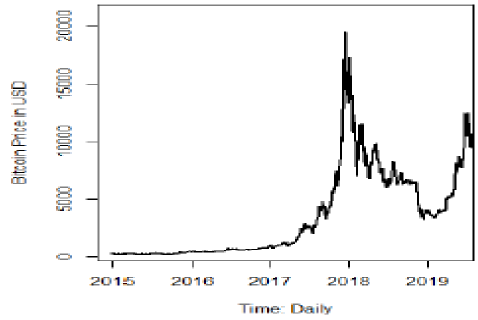

Figure 1b: Bitcoin Price (01-01-15 to 30-06-

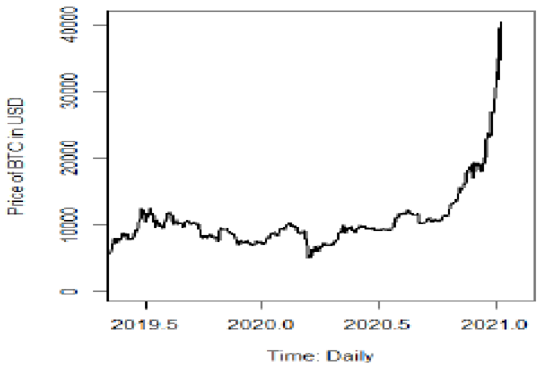

Figure 1c: Bitcoin Price (01-07-19 to 11-01-21)

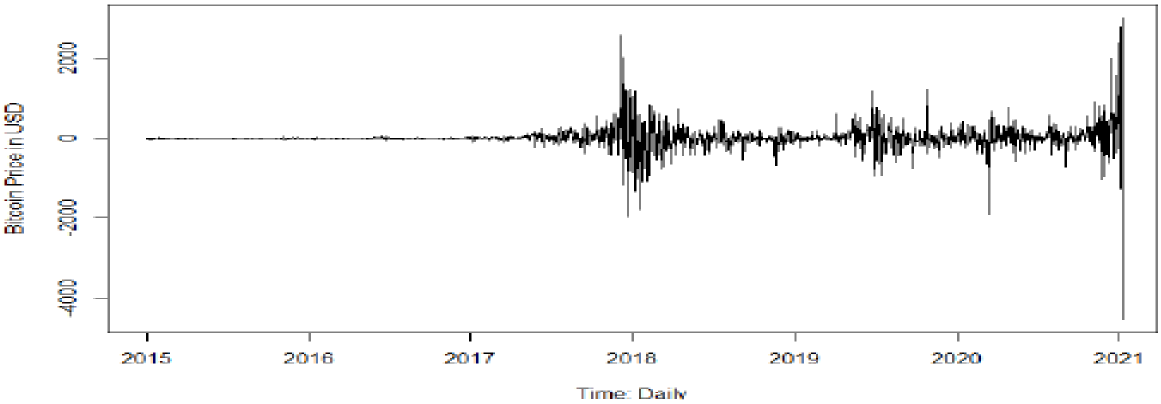

Figure 1d: Bitcoin Price Difference (01-15-21 to 11-01-21)

Figure 1e: Bitcoin Price (Log)

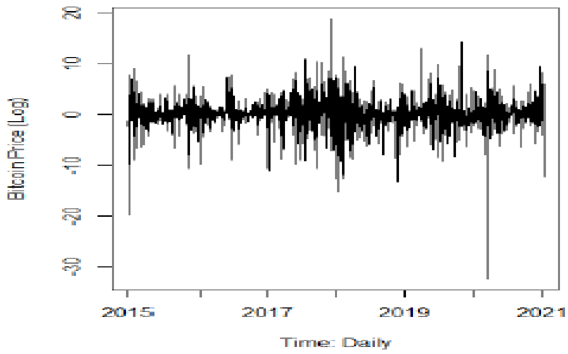

Figure 1f: Bitcoin Price (Log difference)

4.3. Stationarity test

The stationarity result (Table 4) shows that the ADF test accepts the null of non-stationarity, with $\tau_{\mu} > ADF_{\alpha}$ in all the test equations. The ERS statistics for the validation-set (daily) and training-set (weekly) appear stationary but this was refuted when we add the linear trend, hence the ERS nulls are accepted at 1%. The KPSS rejects the test's null of stationarity at 1%. The results confirm non-stationarity for the training, validation and combined data, for all series. The first difference tests are all stationary and significant at 1%, except for the validation-set for monthly series at 5% (no trend) and 10% (linear trend). Bitcoin price for each periodicity is clearly, I (1), and indistinguishable from a random walk.

4.4. Training the Bitcoin price data

We train the daily series for 1642 days (01\01\15 – 30\06\19) and evaluate the models for a validation period of 561 days (01\07\19 – 11\01\2021). The weekly data was trained for 235 weeks (01\01\15 – 27\06\19) and validated for 81 weeks (28\06\19 – 11\01\2021). The monthly series was trained for 54 months (01\01\15 – 01\07\19) and validated for 20 months (01\08\19 – 11\01\2021). For the daily series, the Naïve forecast produces a residual

DOI: 10.25103/ijbesar.153.06

standard error of 220.38. The linear model and its trend coefficients are significant with the model p-value of approximately zero. The ETS (M,Ad,N) parameters reported are $[\alpha(0.9999), \beta(0.1887), \Psi(0.8)]$, with Initial states $[(a = 308.66, b = -3.43)]$, and σ (sigma) = 0.03. The ARIMA (auto) uses the lowest AIC to select an ARIMA (2, 1, 0) while considering the specification's stationarity test. There was an average of about 20 different network specifications in the neural network.

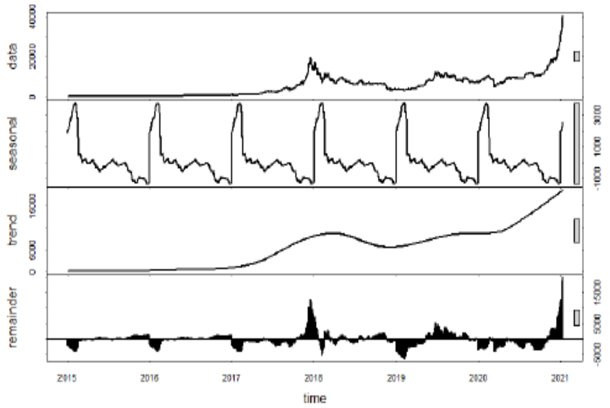

Figure 2a: STL decomposition of Bitcoin price

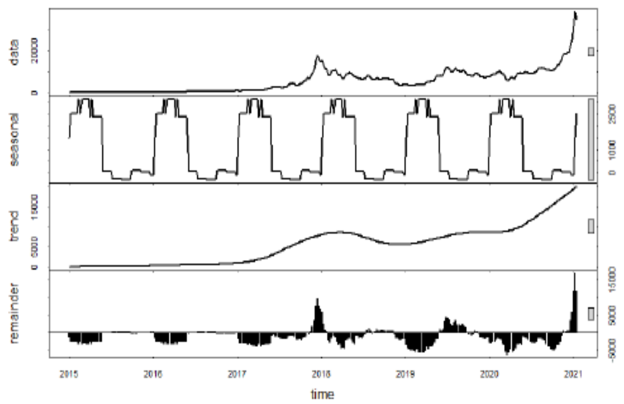

Figure 2b: STL decomposition of Bitcoin price (weekly)

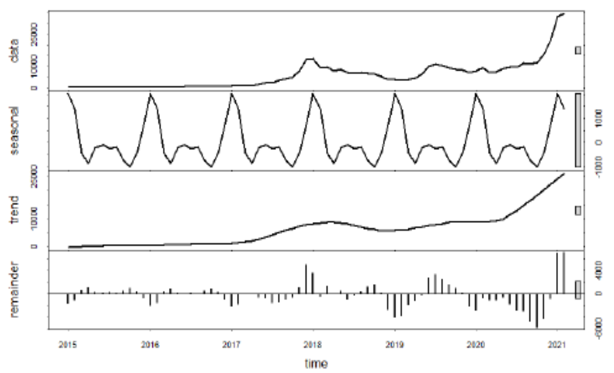

Figure 2c: STL decomposition of Bitcoin price (monthly)

Table 3: STL decomposition statistics

Statistics	Daily			Weekly			Monthly		
	Seasonal	Trend	Random	Seasonal	Trend	Random	Seasonal	Trend	Random
Min.	-1340.10	278.04	-6274.98	-278.75	32.08	-6207.36	-1007.97	-266.34	-5626.28
1st Qu.	-544.17	630.86	-1389.27	-92.46	637.03	-2789.17	-428.82	676.24	-1265.32
Median	-116.95	5863.26	-112.48	113.96	5886.31	-1371.57	-210.77	5980.26	24.66
Mean	102.54	5370.82	-182.97	1034.65	5483.00	-1116.60	46.12	6019.32	-164.46
3rd Qu.	138.14	8493.48	711.59	2435.79	8471.77	142.39	724.53	8505.58	613.74
Max.	3777.14	18402.07	19826.54	3153.41	20124.91	16912.48	2022.29	26240.30	7031.75
IQR	682.30	7862.60	2100.90	2528.00	7835.00	2932.00	1153.00	7829.00	1879.00
IQR%	8.40	97.00	25.90	31.20	96.60	36.10	13.80	93.70	22.50

Qu.: Quartile. IQR: Interquartile range IQR%: Percentage IQR.
Source: (Author's construct, 2023)

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Table 4: Stationarity test

	Level (y_t)						Difference (Δy_t)						
	τ_μ	ADF_α	τ_τ	ERS_α	τ_η	$KPSS_\alpha$	τ_μ	ADF_α	Prob.	τ_τ	ERS_α	τ_η	$KPSS_\alpha$
Daily													
Training	-0.84	-3.43	-0.05	-2.57	3.14	0.74	-26.5	-3.43	0.00	-26.50	-2.57	0.11	0.74
Validation	3.19	-3.34	3.45	-2.57	1.39	0.74	-7.78	-3.44	0.00	-2.13	-1.94	1.18	0.74
Full	-1.43	-3.97	-2.57	3.11	4.27	0.74	-28.3	-3.43	0.00	-11.41	-2.57	0.52	0.74
Training[a]	2.18	3.31	-1.86	-3.48	0.34	0.22	-26.5	-3.96	0.00	-8.32	-3.48	0.07	0.22
Validation[a]	2.97	-3.97	0.42	-3.48	0.53	0.22	-8.47	-3.97	0.00	-3.48	-0.12	0.19	0.22
Full[a]	1.16	-3.96	-0.77	-3.48	0.19	0.22	-28.4	-3.96	0.00	-9.09	-3.48	0.52	0.74
Weekly													
Training	-0.73	-3.46	3.25	-2.58	0.97	0.74	-7.67	-3.46	0.00	-7.61	-2.58	0.10	0.74
Validation	1.92	-3.51	1.30	-2.59	0.66	0.74	-5.99	-3.51	0.00	-5.98	-2.59	0.68	0.74
Full	0.99	-3.45	1.53	-2.57	1.70	0.74	-5.94	-3.45	0.00	-5.35	-2.57	0.41	0.74
Training[a]	-2.36	-4.00	-0.42	-3.50	0.30	0.22	-7.70	-4.00	0.00	-5.48	-3.46	0.06	0.22
Validation[a]	1.13	-4.08	-1.01	-3.65	0.25	0.22	-6.72	-4.08	0.00	-5.53	-3.65	0.14	0.22
Full[a]	-0.75	-3.99	-1.22	-3.47	0.19	0.22	-6.18	-3.99	0.00	-4.14	-3.47	0.15	0.22
Monthly													
Training	-1.24	-3.56	1.38	-2.64	0.57	0.74	-4.68	-3.56	0.00	-4.74	-2.61	0.11	0.74
Validation	2.29	-3.81	-0.63	-2.69	0.48	0.74	-4.53	-3.81	0.00	-2.58	-2.69	0.45	0.74
Full	0.34	-3.52	1.46	-2.60	1.01	0.74	-4.99	-3.52	0.00	-4.97	-2.60	0.35	0.74
Training[a]	-2.59	-4.14	2.23	-3.77	0.19	0.22	-4.66	-4.14	0.00	-4.70	-3.76	0.08	0.22
Validation[a]	0.76	-4.50	-2.08	-3.77	0.18	0.22	-3.49	-4.50	0.07***	-3.27	-3.77	0.14	0.22
Full[a]	-1.60	-4.09	-1.16	-3.67	0.09	0.22	-5.27	-4.09	0.00	-5.33	-3.69	0.13	0.22

ADF_α: MacKinnon one-sided p-values; Elliott-Rothenberg-Stock (ERS_α); Kwiatkowski-Phillips-Schmidt-Shin ($KPSS_\alpha$).
ADF Null (H_0): Nonstationary; DF-GLS Null (H_0): Non-stationary; KPSS, Null (H_0): Stationary
[a]Test has intercept with linear trend, others are with no (time) trend; the Critical Value$(\mathrm{C.V.})$ reported are at 1%;
** stationarity at 5%; *** Stationary at 10%.
Source: (Author's construct, 2023)

The NNAR (4, 1, 3) with σ^2 estimated as 49270 was selected based on test-sample. We estimate Holt-Winters model with trend and without seasonal component, which accommodates for non-multiple of the number of observations. The smoothing parameters and coefficients obtained are [$\alpha(1)$, $\beta(0.022)$, γ(False)] and [a (11636.46) and b (87.49)], respectively. We estimate the weekly and monthly sets and compared the forecast performance with our daily counterparts. Next, we apply these models to predict the price of BTC for the validations periods to shed light on performance. Figure 3a–3g shows the time-series plots of actual and predicted values during training and validation periods for the daily series, while Figure 4a–4g and Figure 5a–5g (appendix) show same for weekly and monthly datasets. Table D.1 (appendix) presents a 40-day (01/07/19–09/08/19) summary of predictions, as well as forecast errors (absolute and percentage) in the validation periods for the daily price of Bitcoin. The table presents the average point forecast, 80%, and 95% intervals for each forecasting method. A cursory look at the table indicates the result favours the Naïve forecast performance – which presents data-frame of lower errors – relative to other predictive measures. The forecast accuracy measures are employed to make appropriate judgment on the best forecast model.

4.5. Forecast accuracy

Table 5 presents the training sample and validation sample forecast performance evaluated with the forecast accuracy measures. When we trained the daily series on each forecast model except for the MPE, four of the accuracy measures [RMSE, MAE, MASE and MAPE] showed that the Naïve model performed better than other predictive models. The Naïve model has the least values for the various measures as indicated [with asterisk *] in Table 5. With the weekly series and using the RMSE, MAE and MAPE as evaluation benchmarks, the Naïve method still outperformed other models. However, the MPE support that the linear model is best and the MAPE indicates that the Exponential

DOI: 10.25103/ijbesar.153.06

smoothing model outperformed others. The monthly series also confirmed the superiority of the Naïve method over others as three of the accuracy measures when we trained with the monthly data shows Naïve method has the lowest forecast error. Turning to the validation samples evaluation, the results supported the HWM's superiority over others for the daily sample, except for the MAPE and MASE measures.

Table 6 presents the result of the DM tests. We compare the accuracy of the forecast performance from two different models under same data frequency. The result is similar to reports in Table 5. Comparing the Naïve model (F_1) to another forecast models (F_2) for each of the data frequency, we confirm that the Naïve model is more accurate in forecasting the test sample price ($p < 5\%$), which is not surprising since a better forecast for BTC price is its last previous price. For all the data frequency, the DM tests confirm the ARIMA superiority over the NNAR in the test-sample periods (Munim *et al.*, 2019). We complete some residuals diagnostic tests to verify the validity of the forecast models (see Table D.2 in the appendix). The Lbox (Q*) statistics suggest the presence of autocorrelation, while the Qm (x^2) test indicates the occurrence of conditional heteroscedasticity, except for the Naïve model.

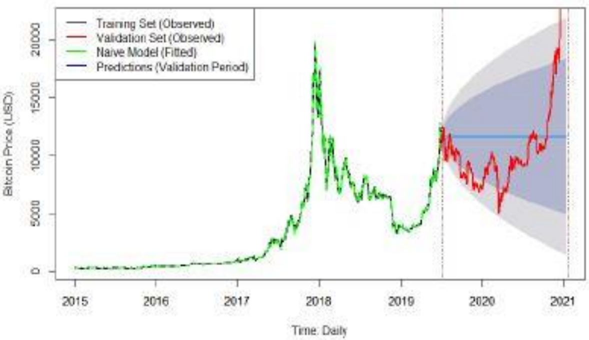

Figure 3a: Prediction in the validation period (Naïve model)

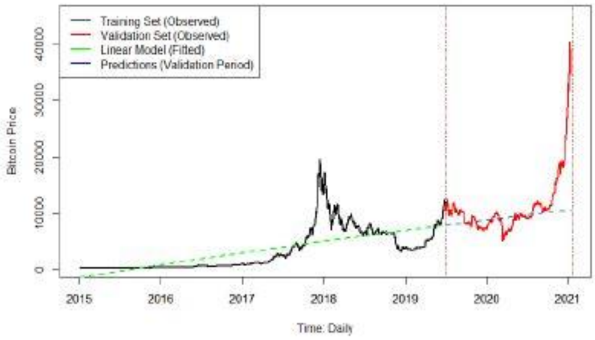

Figure 3b: Prediction in the validation period (Linear model)

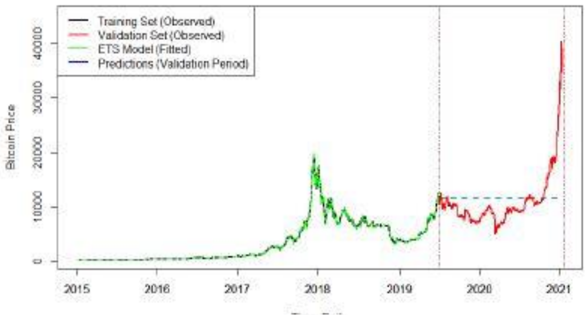

Figure 3c: Prediction in the validation period (ETS model)

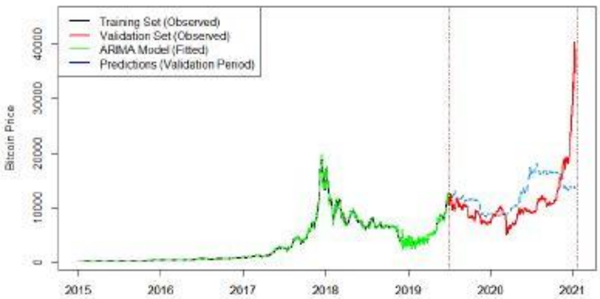

Figure 3d: Prediction in the validation period (ARIMA model)

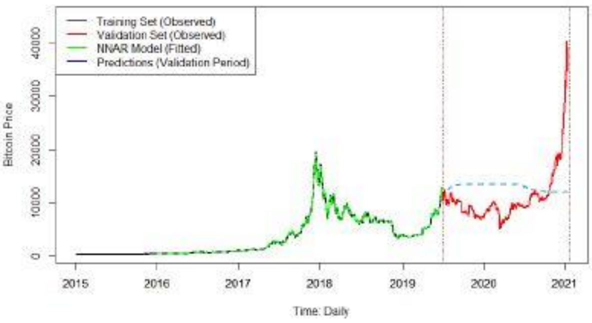

Figure 3e: Prediction in the validation period (NNAR model)

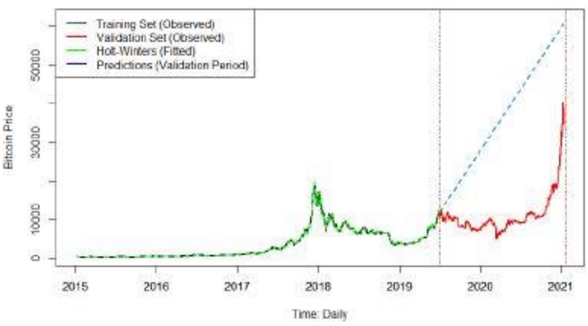

Figure 3f: Prediction in the validation period: (Holt-Winters model)

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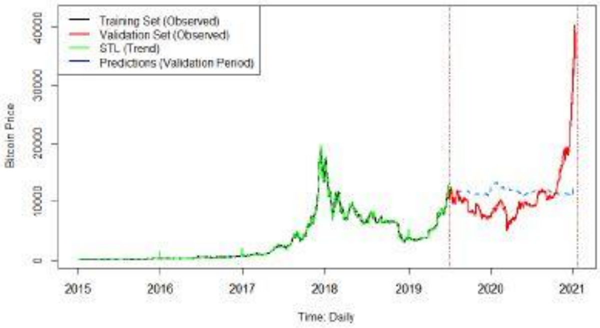

Figure 3g: Prediction in the validation period (STL-trend)

4.6. Future forecasts for Bitcoin price

We predict the out-of-sample forecasts of the price for the daily periodicity. We combine the training and validation periods and estimate the forecast models on the full data. A total of 2203, 316 and 75 observations are applied for the daily, weekly and monthly series, respectively. The forecast periods follow directly behind the closing of validation periods starting January 12, 2021. The forecast horizon does not exceed the validation periods. We compare the prediction intervals for different models with different levels and decreasing certainty for varying future depicted by the prediction cone (Figure 6a–6g). The figures show the future forecasts for daily Bitcoin price. Table 7 shows 40-days forecasts and future returns (percentage) for the daily price. The value of the last observed day Bitcoin price ($34,662.48) was used to calculate the static percent increase for the 40 days (12-01-2021– 20-02-2021).

4.7. Is the Bitcoin price forecast sensitive to the choice of data frequency?

We check whether forecasting the price is sensitive to the data choice. The forecast accuracy result obtained is presented in Table 5. For the weekly series, the linear is superior except for the MPE measures, indicating that the HWM outperforms others. In contrast, for the monthly series, the Naïve method outperforms others except for the MPE that shows the ETS is superior. Comparing with daily series, we conclude that frequency matters in forecasting the Bitcoin price series. Overall, the results of the model comparison tests (Table 6) establish that irrespective of the data frequency, the Naïve model is superior and more accurately predicts the price than others. The DM test is sufficient to submit that forecasting the price is not sensitive to the periodicity.

Table 5: Training-sample and validation-sample forecast performance

	Training					Validation				
Forecast Methods	RMSE ($)	MAE ($)	MPE (%)	MAPE (%)	MASE (I)	RMSE ($)	MAE ($)	MPE (%)	MAPE (%)	MASE (I)
Daily										
NAIVE	204.83*	87.451*	0.145	1.990*	0.03*	4993.43	3184.47	-16.70	28.75	0.94*
LINEAR	2591.4	1767.5	-16.96**	144.0	0.52	4900.70	2540.04	6.593	18.46*	0.95
ESM	215.07	91.292	0.112	2.050	0.03	5011.04	3276.57	-18.42	29.98	0.96
ARIMA	281.55	133.23	0.071	2.790	0.04	5032.21	3607.43	-23.14	32.08	1.06
NNAR	221.97	108.72	0.432	2.230	0.03	5769.21	4492.51	-33.83	44.52	1.32
STL	228.75	107.80	0.280	5.430	0.03	5069.70	3424.49	-19.72	31.56	1.01
HWM	220.31	92.241	0.121	2.060	0.03	28056.3**	25298.4**	-250.8**	250.8	7.44
Weekly										
NAIVE	479.74*	242.38*	0.752	6.610	0.07*	6049.76	4670.00	-33.58	44.00	1.38
LINEAR	2552.6	1748.6	-16.43**	142.0	0.52	5643.41**	2786.10**	6.512	18.48**	0.83**
ESM	589.89	270.74	0.310	6.280*	0.08	25218.2	22953.2	-225.5	225.5	6.80
ARIMA	695.08	368.39	0.472	8.350	0.11	6246.72	4975.46	-39.21	45.76	1.48
NNAR	532.01	302.70	-1.68	6.660	0.09	8335.71	6578.63	-28.16	58.98	1.95
STL	615.87	328.21	1.701	19.90	0.10	7649.24	6951.81	-65.75	72.24	2.06
HWM	591.38	267.27	0.940	6.240	0.08	36425.1	32865.4	-316.4*	316.4	9.74
Monthly										
NAIVE	1210.5	636.69*	4.061	13.54*	0.19*	8010.76	4502.49	-4.320	27.71	1.32

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LINEAR	2464.3	1784.5	-16.28*	141.6	0.52	7991.66**	4103.34**	12.17	21.19*	1.20**
ESM	1316.8	657.19	2.703	14.00	0.20	19550.7	17722.8	-167.3**	167.3	5.20**
ARIMA	1764.3	965.43	1.971	18.32	0.28	7451.33	4754.54	-13.73	31.21	1.39
NNAR	1083.7*	694.43	-6.571	20.85	0.20	9501.09	5416.06	28.82	29.33	1.59
STL	1218.9	787.38	10.33	61.87	0.23	8411.28	4519.36	10.44	24.68	1.32
HWM	1305.3	648.19	9.363	16.52	0.19	7822.63	4268.52	-1.334	25.29	1.25

MASE is an index (I) which compares a chosen model predictive performance (for instance, the MPE) to the naive forecast on the training set. The index value less than 1 indicates that the compared model has a lower average error than naïve forecasts (in the training period). If the index value is higher than 1, it indicates poor performance relative to (training period) naive forecasts.
* Naïve method better.
** Other model outperformed naïve.
Source: (Author's construct, 2023)

Table 6: The Diebold-Mariano (DM) test for test-sample

$F_1(\downarrow)$ $F_2(\rightarrow)$	ETS		LINEAR		ARIMA		NNAR		STL		HWM	
Daily												
NAIVE	7.62	(4.34×10^{-14})	-14.02	(2.20×10^{-16})	-6.42	(1.75×10^{-10})	-7.52	(3.97×10^{-14})	-8.97	(2.20×10^{-16})	-7.38	(2.52×10^{-13})
ETS			-14.03	(2.20×10^{-16})	-9.94	(2.20×10^{-12})	-9.03	(2.20×10^{-16})	-7.88	(6.48×10^{-15})	-7.38	(2.48×10^{-13})
LINEAR					13.95	(2.20×10^{-16})	15.74	(2.20×10^{-16})	-8.14	(7.79×10^{-16})	-5.52	(3.96×10^{-8})
ARIMA							15.74	(2.20×10^{-16})	-8.14	(7.95×10^{-16})	-7.38	(2.51×10^{-13})
NNAR									-8.96	(2.20×10^{-16})	-7.38	(2.56×10^{-13})
STL											-7.38	(2.47×10^{-13})
Weekly												
NAIVE	4.21	(3.66×10^{-14})	-5.42	(1.45×10^{-7})	-3.44	(0.0007)	2.59	(0.0052)	4.20	(3.83×10^{-5})	-1.41	(0.1612)
ETS			-5.50	(1.02×10^{-7})	-3.44	(0.0006)	-5.53	(8.53×10^{-8})	4.20	(3.83×10^{-5})	-3.65	(0.0003)
LINEAR				(1.02×10^{-7})	5.21	(4.07×10^{-7})	6.09	(4.67×10^{-9})	4.19	(3.91×10^{-5})	5.42	(1.46×10^{-7})
ARIMA							3.93	(0.0001)	4.20	(3.85×10^{-5})	5.42	(1.46×10^{-7})
NNAR									4.20	(3.81×10^{-5})	-1.85	(0.0661)
STL											4.20	(3.87×10^{-5})
Monthly												
NAIVE	2.77	(7.79×10^{-5})	-2.76	(0.0079)	-2.13	(0.0374)	2.05	(0.0453)	2.22	(0.0308)	-2.24	(0.0495)
ETS			-3.16	(0.0002)	-3.25	(0.0020)	-3.34	(0.0015)	-2.06	(0.0443)	-2.37	(0.0214)
LINEAR					1.93	(0.0589)	2.66	(0.0031)	2.08	(0.0429)	2.41	(0.0194)
ARIMA							2.05	(0.0453)	2.04	(0.0464)	-2.35	(0.0150)
NNAR									-2.35	(0.0225)	-2.26	(0.0281)
STL											-2.32	(0.0193)

DM test compares two forecast models $[F_1, F_2]$. It shows whether (F_1) is more accurate than model (F_2). The test is based on the loss differentials, $d_t = L(e_{1,t}) - L(e_{2,t})$. $H_0: E[d_t] = 0$ (F_1 is same as F_2) and $H_1: E[d_t] \neq 0$. Assume $e_{j,t} = \hat{y}_{T+h|T} - y_t$, sample mean loss differential $\overline{d} = T^{-1} \sum_{t=1}^{T} [L(e_{1,t}) - L(e_{2,t})$ and DM statistic $(DM_\alpha) = \overline{d}/\sqrt{T^{-1}[2\pi \hat{f}(0)]} \rightarrow d \sim n(O,1)$, where $2\pi \hat{f}(0)$ is a consistent estimator of the asymptotic variance, $\sqrt{T d}$. Since DM_α converge to a normal distribution, H_0 is rejected at 5% if $|DM_\tau| > 1.96$, but cannot be rejected, if $|DM_\tau| \leq 1.96$. Probability *(p)* < 0.05 indicates that F_1 is better. Figure in the parenthesis indicate p–value, others are DM_τ. Source: (Author's construct, 2023)

DOI: 10.25103/ijbesar.153.06

	Future forecasts for daily price							Percent increase from previous daily price (daily returns)						
Date	NAIVE	ETS	LINEAR	ARIMA	NNAR	STL	HWM	NAIVE	ETS	LIN.	ARIMA	NNAR	STL	HWM
12/1/21	35228.94	34255.39	31581.96	32919.03	33815.88	34104.02	34994.37	1.63	-1.17	-8.89	-5.03	-2.44	-1.61	0.96
13/1/21	35463.58	33929.33	31604.16	33131.23	35419.89	33656.85	35326.26	0.67	-0.95	0.07	0.64	4.74	-1.31	0.95
14/1/21	35643.62	33668.48	31626.37	33636.11	37077.15	33299.11	35658.15	0.51	-0.77	0.07	1.52	4.68	-1.06	0.94
15/1/21	35795.41	33459.80	31648.59	33477.27	37614.15	33199.07	35990.04	0.43	-0.62	0.07	-0.47	1.45	-0.30	0.93
16/1/21	35929.13	33292.86	31670.81	33507.14	39106.08	32952.50	36321.92	0.37	-0.50	0.07	0.09	3.97	-0.74	0.92
17/1/21	36050.03	33159.30	31693.04	33642.51	37202.53	32769.34	36653.81	0.34	-0.40	0.07	0.40	-4.87	-0.56	0.91
18/1/21	36161.20	33052.45	31715.27	33628.89	35287.09	32622.81	36985.70	0.31	-0.32	0.07	-0.04	-5.15	-0.45	0.91
19/1/21	36264.68	32966.98	31737.51	33390.49	36993.15	32487.96	37317.59	0.29	-0.26	0.07	-0.71	4.83	-0.41	0.90
20/1/21	36361.87	32898.59	31759.77	33373.33	39825.69	32518.07	37649.48	0.27	-0.21	0.07	-0.05	7.66	0.09	0.89
21/1/21	36453.79	32843.89	31782.02	33444.91	40162.88	32443.04	37981.37	0.25	-0.17	0.07	0.21	0.85	-0.23	0.88
22/1/21	36541.22	32800.12	31804.29	33287.60	40610.10	32383.02	38313.26	0.24	-0.13	0.07	-0.47	1.11	-0.18	0.87
23/1/21	36624.76	32765.11	31826.56	33115.95	41528.43	32458.89	38645.15	0.23	-0.11	0.07	-0.52	2.26	0.23	0.87
24/1/21	36704.89	32737.10	31848.84	33118.52	38265.45	32614.55	38977.03	0.22	-0.09	0.07	0.01	-7.86	0.48	0.86
25/1/21	36781.99	32714.69	31871.12	33151.03	38425.51	32583.82	39308.92	0.21	-0.07	0.07	0.10	0.42	-0.09	0.85
26/1/21	36856.38	32696.76	31893.42	33444.37	39854.05	32559.24	39640.81	0.20	-0.05	0.07	0.88	3.72	-0.08	0.84
27/1/21	36928.33	32682.42	31915.72	33780.11	40211.47	32733.64	39972.70	0.20	-0.04	0.07	1.00	0.90	0.54	0.84
28/1/21	36998.07	32670.95	31938.03	34064.71	39049.12	32855.33	40304.59	0.19	-0.04	0.07	0.84	-2.89	0.37	0.83
29/1/21	37065.78	32661.77	31960.34	34086.94	41098.66	32842.75	40636.48	0.18	-0.03	0.07	0.07	5.25	-0.04	0.82
30/1/21	37131.63	32654.43	31982.66	34140.52	41179.34	32832.68	40968.37	0.18	-0.02	0.07	0.16	0.20	-0.03	0.82
31/1/21	37195.78	32648.55	32004.99	34086.56	38673.54	32962.05	41300.25	0.17	-0.02	0.07	-0.16	-6.09	0.39	0.81
01/2/21	37258.34	32643.85	32027.33	34079.07	39086.09	32997.06	41632.14	0.17	-0.01	0.07	-0.02	1.07	0.11	0.80
02/2/21	37319.43	32640.09	32049.67	34098.15	40936.35	32991.91	41964.03	0.16	-0.01	0.07	0.06	4.73	-0.02	0.80
03/2/21	37379.14	32637.09	32072.02	33965.90	40562.40	32987.78	42295.92	0.16	-0.01	0.07	-0.39	-0.91	-0.01	0.79
04/2/21	37437.57	32634.68	32094.38	34069.73	40843.01	33025.94	42627.81	0.16	-0.01	0.07	0.31	0.69	0.12	0.78
05/2/21	37494.79	32632.76	32116.75	34381.01	41788.23	33034.37	42959.70	0.15	-0.01	0.07	0.91	2.31	0.03	0.78
06/2/21	37550.88	32631.22	32139.12	34482.50	40013.55	33032.26	43291.59	0.15	0.00	0.07	0.30	-4.25	-0.01	0.77
07/2/21	37605.91	32629.98	32161.50	34503.17	39536.39	33030.57	43623.47	0.15	0.00	0.07	0.06	-1.19	-0.01	0.77
08/2/21	37659.92	32629.00	32183.89	34668.17	40953.30	33040.29	43955.36	0.14	0.00	0.07	0.48	3.58	0.03	0.76
09/2/21	37712.97	32628.21	32206.28	34742.22	40916.83	32941.29	44287.25	0.14	0.00	0.07	0.21	-0.09	-0.30	0.76
10/2/21	37765.12	32627.58	32228.68	34652.02	39794.47	32940.42	44619.14	0.14	0.00	0.07	-0.26	-2.74	0.00	0.75
11/2/21	37816.41	32627.07	32251.09	34986.39	41303.55	32939.73	44951.03	0.14	0.00	0.07	0.96	3.79	0.00	0.74
12/2/21	37866.88	32626.67	32273.50	35019.51	41776.01	32841.25	45282.92	0.13	0.00	0.07	0.09	1.14	-0.30	0.74
13/2/21	37916.56	32626.35	32295.93	34939.95	40103.16	32445.97	45614.81	0.13	0.00	0.07	-0.23	-4.00	-1.20	0.73
14/2/21	37965.50	32626.09	32318.36	34896.87	41021.09	32445.61	45946.70	0.13	0.00	0.07	-0.12	2.29	0.00	0.73
15/2/21	38013.72	32625.88	32340.79	34608.77	42168.37	32445.33	46278.58	0.13	0.00	0.07	-0.83	2.80	0.00	0.72
16/2/21	38061.26	32625.72	32363.24	34514.60	40757.48	32050.26	46610.47	0.13	0.00	0.07	-0.27	-3.35	-1.22	0.72
17/2/21	38108.14	32625.58	32385.69	34548.77	40939.21	32050.08	46942.36	0.12	0.00	0.07	0.10	0.45	0.00	0.71
18/2/21	38154.39	32625.48	32408.15	34702.49	42078.47	30888.73	47274.25	0.12	0.00	0.07	0.44	2.78	-3.62	0.71
19/2/21	38200.04	32625.39	32430.61	34313.74	40755.05	30888.61	47606.14	0.12	0.00	0.07	-1.12	-3.15	0.00	0.70
20/2/21	38245.10	32625.33	32453.09	34361.77	40330.67	29727.31	47938.03	0.12	0.00	0.07	0.14	-1.04	-3.76	0.70

DOI: 10.25103/ijbesar.153.06

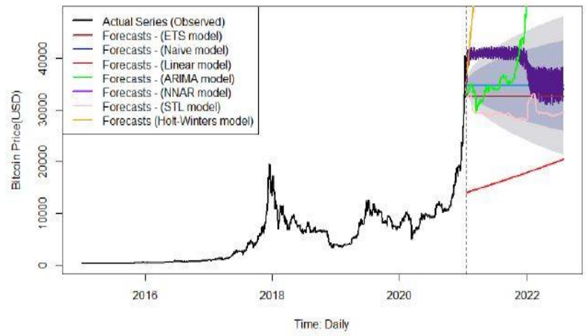

Figure 6a: Daily Bitcoin future forecasts (Naïve model)

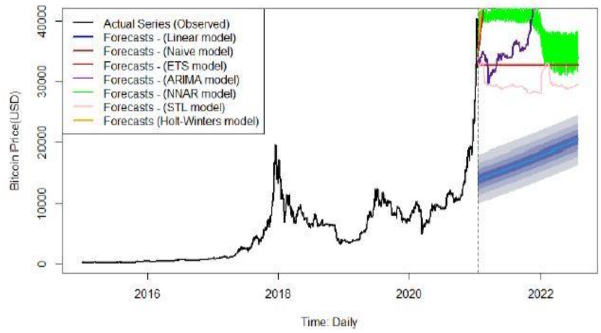

Figure 6b: Daily Bitcoin future forecasts (Linear model)

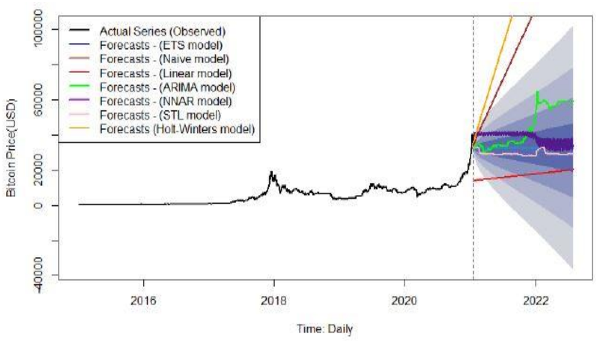

Figure 6c: Daily Bitcoin future forecasts (ETS model)

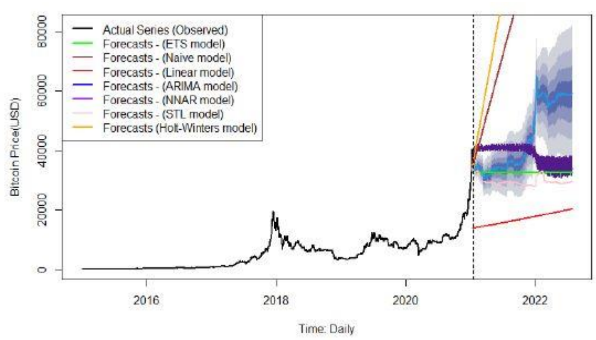

Figure 6d: Daily Bitcoin future forecasts (ARIMA model)

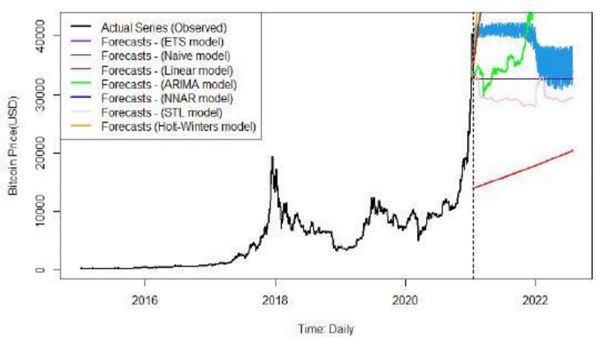

Figure 6e: Daily Bitcoin future forecasts (NNAR model)

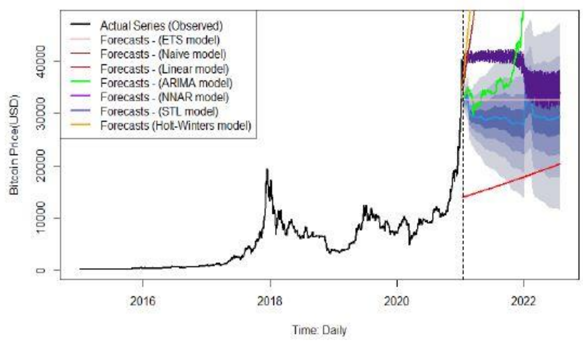

Figure 6f: Daily Bitcoin future forecasts (STL)

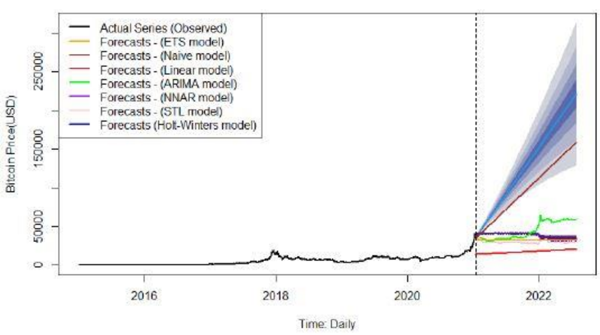

Figure 6f: Daily Bitcoin future forecasts (Holt-Winter model)

DOI: 10.25103/ijbesar.153.06

5. Conclusions

The study aims to compare the outcome of statistical and machine learning models, and to verify how the different periodicity of the Bitcoin price series, including daily, weekly and monthly, performs in the forecast. We completed forecast models using the Naïve, Linear, Exponential Smoothing, ARIMA, Neural Network, STL and Holt-Winters filters, and apply the standard measures to evaluate the forecast accuracy. The results indicate that the Naïve model provides more accurate performance for the daily series, while the linear model outperforms for both weekly and monthly series. Using the DM statistics to check the forecast equality of the model, the evidence shows that forecasting Bitcoin price is not sensitive to the data periodicity.

The findings have some significant implications. First, because of information asymmetric, increasing economic uncertainties and other markets dynamics, adopting forecast models to predict the directions of Bitcoin price is vital. Second, since Bitcoin has now attracted different stakeholders, including institutional investors, the forecast models of Bitcoin price would serve as guides to make informed decisions in the cryptocurrency markets. Accurate prediction would offer warnings signals to investors, traders and other users in order to circumvent or at least minimize potential-risks due to excessive volatility. Third, the models have implications to drive asset allocations. Asset managers may want to avoid losses by adopting the least error model to predict the likely direction and value of bitcoin price. In periods where volatility is excessive, and the outcome of forecast models becomes sensitive to changes in the training sets, managers may switch funds to invest in financial market assets.

The study has two major limitations: first, for the different periodicity, we apply only the actual price series, and not the returns. Since actual data is usually noisy and may increase the risk of over predictions, we suppose future research can consider other transformation, involving using logarithm or even log-returns. Second, we do not consider the issue of intraday trading. By so doing, we have ignored to convert the models to a trading strategy, which can be compared to possible Monte Carlo of trading strategies where the buy/sell decisions are completely random.

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Appendix

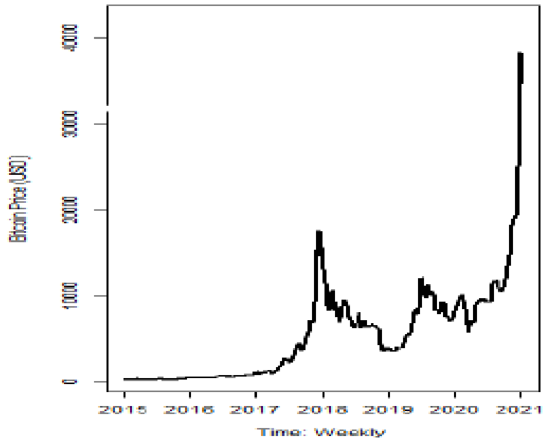

Figure W1: Weekly BTC Price in USD

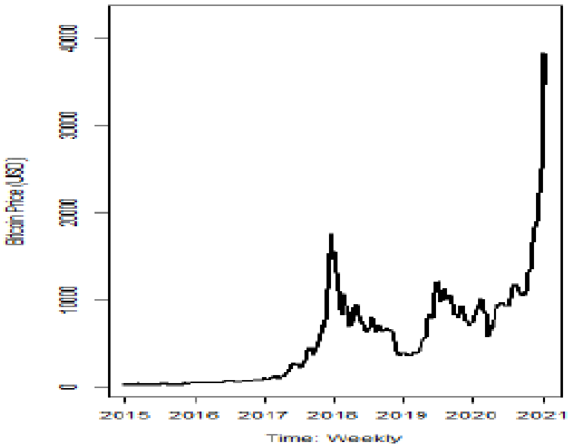

Figure W2: BTC Price 01-01-15 – 27-06-19

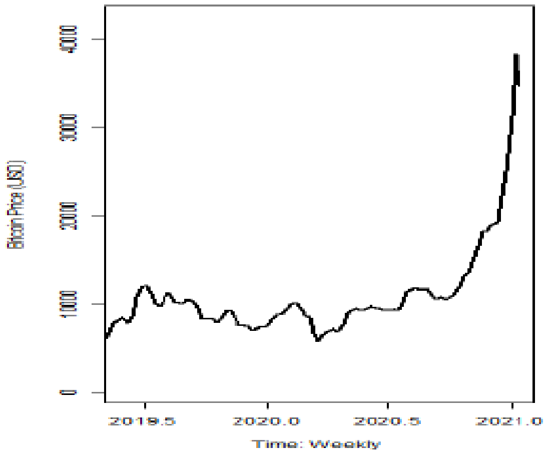

Figure W3: BTC Price 04-07-19 – 11-01-21

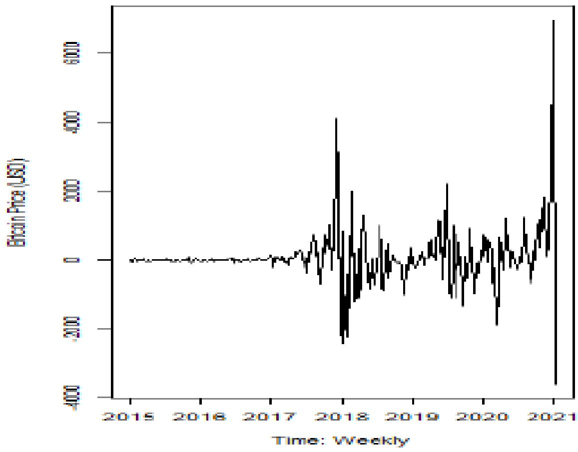

Figure W4: Weekly BTC Price (Difference)

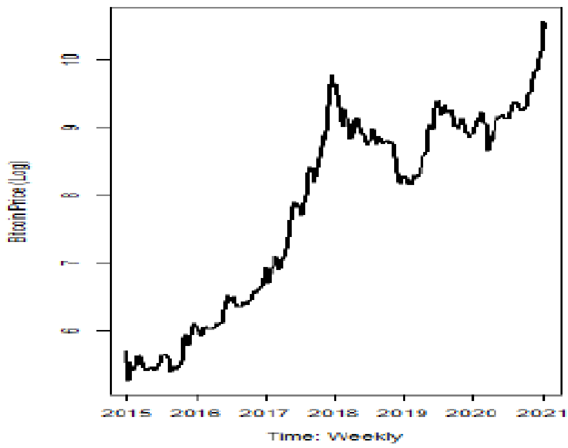

Figure W5: Log of Weekly BTC Price

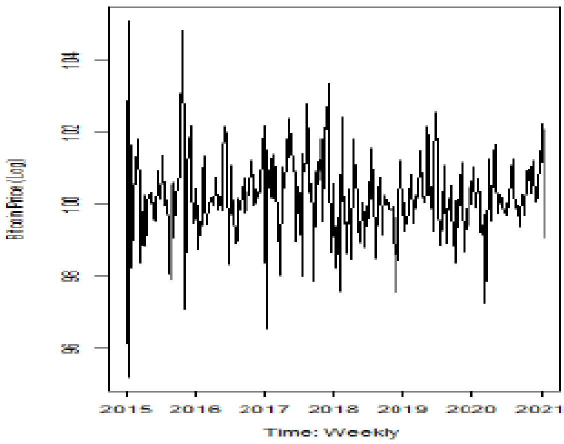

Figure W6: Log of Weekly BTC Price (Difference)

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DOI: 10.25103/ijbesar.153.06

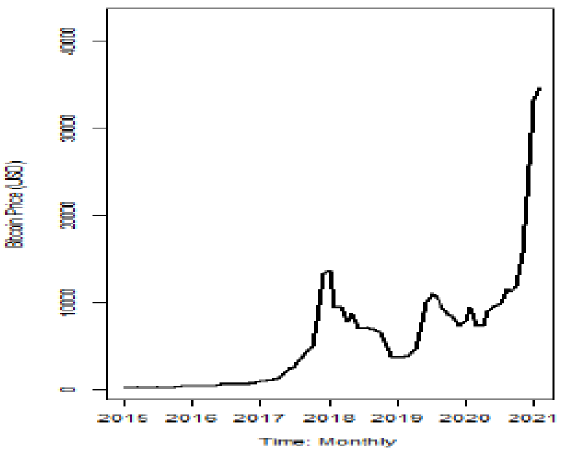

Figure M1: Monthly BTC Price in USD

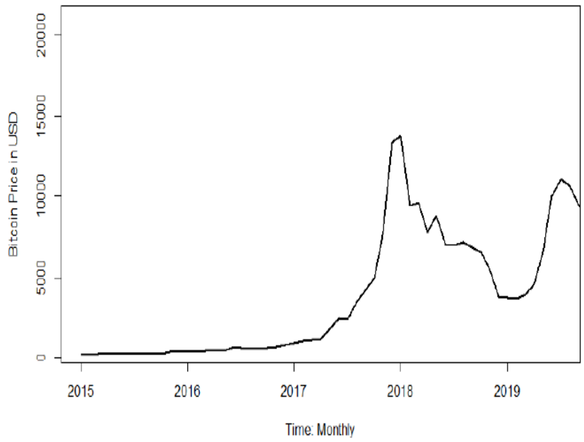

Figure M2: BTC Price 01-15 – 06-19

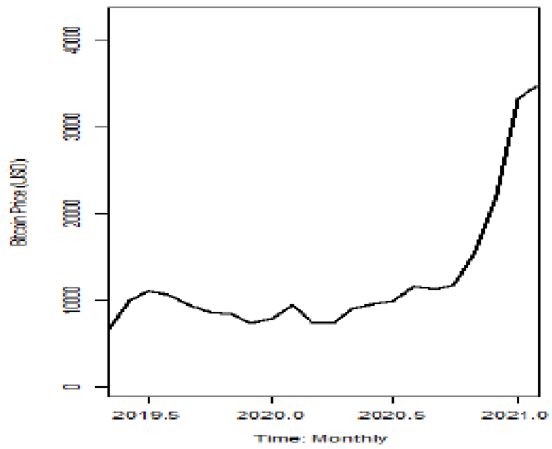

Figure M3: Monthly BTC Price 07-19 – 01-21

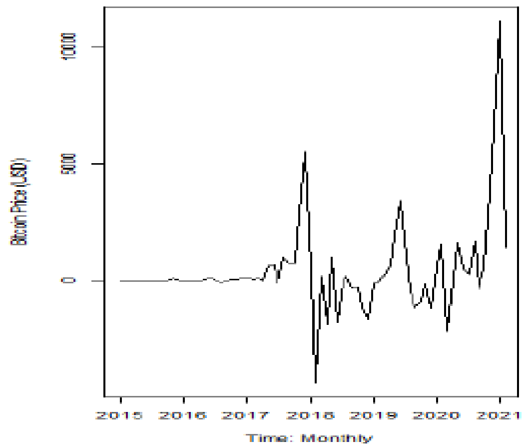

Figure M4: Monthly BTC Price (Difference)

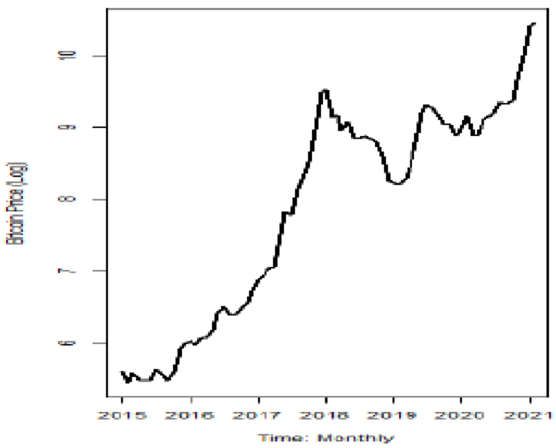

Figure M5: Log of Monthly BTC Price

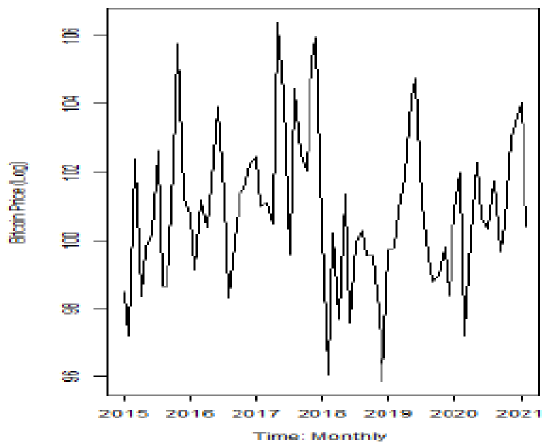

Figure M6: Log of Monthly BTC Price (Difference)

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	PREDICTIONS								PREDICTION ERROR (%)						
Date	ACTUAL	NAIVE	LINEAR	ETS	ARIMA	NNAR	STL	HWM	NAIVE	LINEAR	ETS	ARIMA	NNAR	STL	HWM
01/07/19	11636.50	-	7836.60	11615.70	11307.10	11390.80	11590.90	11724.00	-2.29	-32.65	-0.18	-2.83	-2.11	-0.18	0.75
02/07/19	10697.60	11636.50	7841.70	11599.00	11321.70	11394.30	11631.30	11811.40	5.91	-26.70	8.43	5.83	6.51	8.43	10.41
03/07/19	10412.90	10697.60	7846.70	11585.70	11466.90	11429.30	11602.10	11898.90	9.01	-24.64	11.26	10.12	9.76	11.26	14.27
04/07/19	11201.50	10412.90	7851.80	11575.00	11507.30	11468.90	11578.70	11986.40	1.46	-29.90	3.33	2.73	2.39	3.33	7.01
05/07/19	11715.10	11201.50	7856.80	11566.50	11552.80	11518.10	11599.50	12073.90	-2.97	-32.93	-1.27	-1.39	-1.68	-1.27	3.06
06/07/19	11157.90	11715.10	7861.80	11559.60	11544.30	11571.60	11624.00	12161.40	1.87	-29.54	3.60	3.46	3.71	3.60	8.99
07/07/19	11195.40	11157.90	7866.90	11554.20	11622.80	11626.10	11612.10	12248.90	1.52	-29.73	3.21	3.82	3.85	3.21	9.41
08/07/19	11302.70	11195.40	7871.90	11549.80	11749.10	11680.80	11602.50	12336.40	0.95	-30.35	2.19	3.95	3.35	2.19	9.15
09/07/19	11728.60	11302.70	7876.90	11546.30	11697.90	11735.00	11638.00	12423.90	-2.75	-32.84	-1.55	-0.26	0.05	-1.55	5.93
10/07/19	12432.20	11728.60	7882.00	11543.50	11527.80	11787.80	11675.00	12511.40	-8.27	-36.60	-7.15	-7.27	-5.18	-7.15	0.64
11/07/19	12470.70	12432.20	7887.00	11541.30	11287.40	11838.60	11670.10	12598.90	-8.78	-36.76	-7.45	-9.49	-5.07	-7.45	1.03
12/07/19	11814.40	12470.70	7892.00	11539.50	11228.70	11887.90	11666.20	12686.40	-3.71	-33.20	-2.33	-4.96	0.62	-2.33	7.38
13/07/19	11479.60	11814.40	7897.10	11538.00	11164.60	11935.80	11717.90	12773.90	-0.69	-31.21	0.51	-2.74	3.97	0.51	11.27
14/07/19	11521.20	11479.60	7902.10	11536.90	11171.30	11982.80	11770.20	12861.40	-0.98	-31.41	0.14	-3.04	4.01	0.14	11.63
15/07/19	11022.30	11521.20	7907.10	11536.00	11229.50	12029.00	11768.20	12948.80	3.37	-28.26	4.66	1.88	9.13	4.66	17.48
16/07/19	10434.20	11022.30	7912.20	11535.30	11403.90	12075.00	11766.60	13036.30	9.14	-24.17	10.55	9.29	15.73	10.55	24.94
17/07/19	10447.10	10434.20	7917.20	11534.70	11855.60	12121.50	11767.60	13123.80	8.92	-24.22	10.41	13.48	16.03	10.41	25.62
18/07/19	9532.50	10447.10	7922.20	11534.20	12295.50	12168.60	11769.00	13211.30	19.57	-16.89	21.00	28.99	27.65	21.00	38.59
19/07/19	9937.40	9532.50	7927.20	11533.80	12308.00	12215.10	11768.10	13298.80	14.63	-20.23	16.06	23.86	22.92	16.06	33.83
20/07/19	10533.50	9937.40	7932.30	11533.50	12380.40	12260.70	11767.50	13386.30	8.12	-24.69	9.49	17.53	16.40	9.49	27.08
21/07/19	10675.30	10533.50	7937.30	11533.30	12269.50	12305.00	11838.00	13473.80	6.66	-25.65	8.04	14.93	15.27	8.04	26.22
22/07/19	10669.70	10675.30	7942.30	11533.10	12365.10	12348.30	11908.60	13561.30	6.87	-25.56	8.09	15.89	15.73	8.09	27.10
23/07/19	10467.90	10669.70	7947.30	11532.90	12450.40	12390.60	11908.30	13648.80	9.02	-24.08	10.17	18.94	18.37	10.17	30.39
24/07/19	10192.40	10467.90	7952.40	11532.80	12867.40	12432.00	11908.00	13736.30	11.59	-21.98	13.15	26.25	21.97	13.15	34.77
25/07/19	9803.60	10192.40	7957.40	11532.70	13212.90	12472.40	11872.90	13823.80	15.94	-18.83	17.64	34.78	27.22	17.64	41.01
26/07/19	9912.40	9803.60	7962.40	11532.60	13033.90	12511.60	11837.70	13911.30	14.69	-19.67	16.34	31.49	26.22	16.34	40.34
27/07/19	9849.20	9912.40	7967.40	11532.60	12936.20	12549.50	11837.60	13998.80	15.48	-19.11	17.09	31.34	27.42	17.09	42.13
28/07/19	9816.70	9849.20	7972.40	11532.50	13086.20	12586.20	11837.50	14086.20	16.11	-18.79	17.48	33.31	28.21	17.48	43.49
29/07/19	9439.80	9816.70	7977.50	11532.50	13125.00	12621.60	11781.10	14173.70	20.79	-15.49	22.17	39.04	33.71	22.17	50.15
30/07/19	9567.60	9439.80	7982.50	11532.40	13043.50	12655.80	11724.70	14261.20	19.08	-16.57	20.54	36.33	32.28	20.54	49.06
31/07/19	9553.80	9567.60	7987.50	11532.40	12938.50	12688.80	11724.60	14348.70	19.07	-16.39	20.71	35.43	32.81	20.71	50.19
01/08/19	9762.60	9553.80	7992.50	11532.40	12599.60	12720.70	11724.50	14436.20	16.64	-18.13	18.13	29.06	30.30	18.13	47.87
02/08/19	10148.80	9762.60	7997.50	11532.40	12542.10	12751.40	11682.20	14523.70	12.01	-21.20	13.63	23.58	25.64	13.63	43.11
03/08/19	10477.00	10148.80	8002.60	11532.40	12402.90	12781.00	11639.90	14611.20	8.74	-23.62	10.07	18.38	21.99	10.07	39.46
04/08/19	10656.50	10477.00	8007.60	11532.40	12225.40	12809.40	11639.90	14698.70	6.80	-24.86	8.22	14.72	20.20	8.22	37.93
05/08/19	10817.00	10656.50	8012.60	11532.40	11943.50	12836.70	11639.90	14786.20	5.19	-25.93	6.61	10.41	18.67	6.61	36.69
06/08/19	11272.20	10817.00	8017.60	11532.30	11958.60	12862.90	11651.20	14873.70	1.12	-28.87	2.31	6.09	14.11	2.31	31.95
07/08/19	11792.00	11272.20	8022.60	11532.30	11869.70	12888.10	11662.50	14961.20	-3.44	-31.97	-2.20	0.66	9.29	-2.20	26.88
08/08/19	11649.00	11792.00	8027.60	11532.30	11491.90	12912.10	11662.50	15048.70	-2.31	-31.09	-1.00	-1.35	10.84	-1.00	29.18
09/08/19	11829.90	11649.00	8032.60	11532.30	11312.10	12935.00	11662.50	15136.20	-3.50	-32.10	-2.52	-4.38	9.34	-2.52	27.95

DOI: 10.25103/ijbesar.153.06

Weekly predictions in the validation period

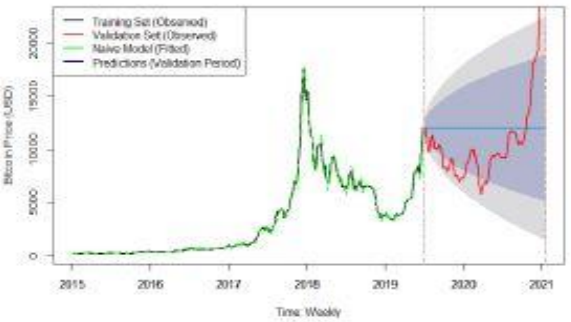

Figure 4a: Weekly predictions in the validation period (Naïve model)

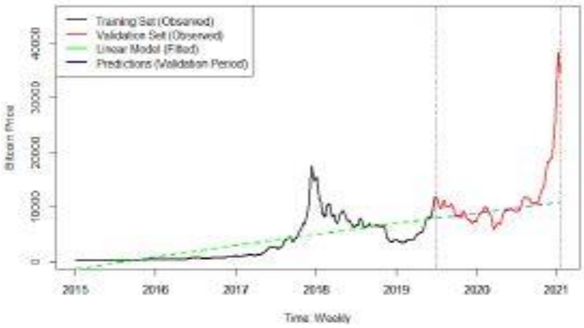

Figure 4b: Weekly predictions in the validation period (Linear model)

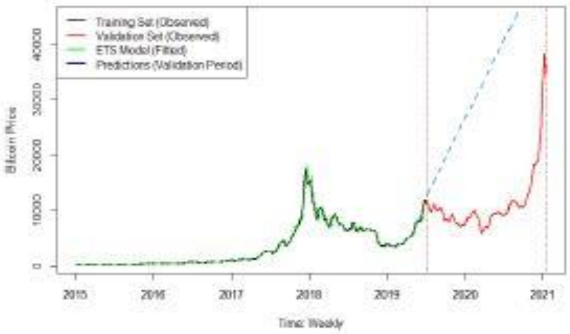

Figure 4c: Weekly predictions in the validation period (ETS model)

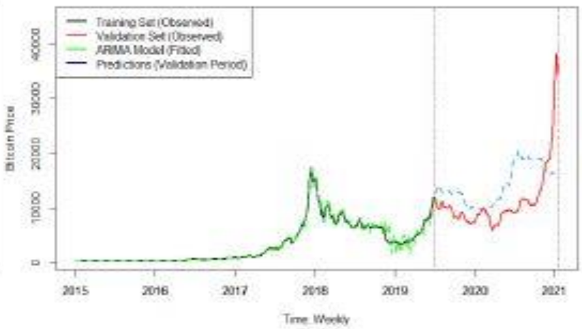

Figure 4d: Weekly predictions in the validation period (ARIMA model)

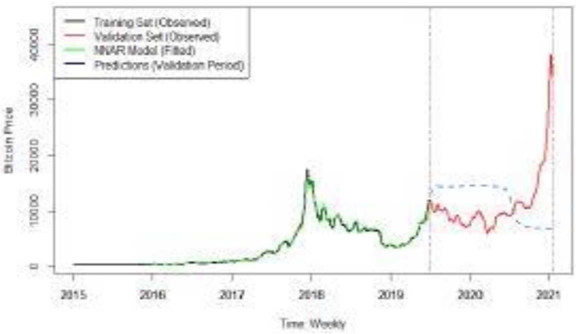

Figure 4e: Weekly predictions in the validation period (NNAR model)

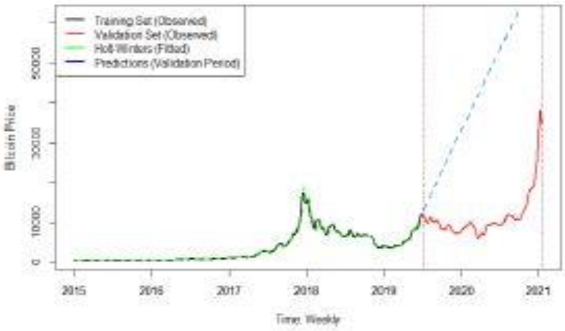

Figure 4f: Weekly predictions in the validation period (HWF model)

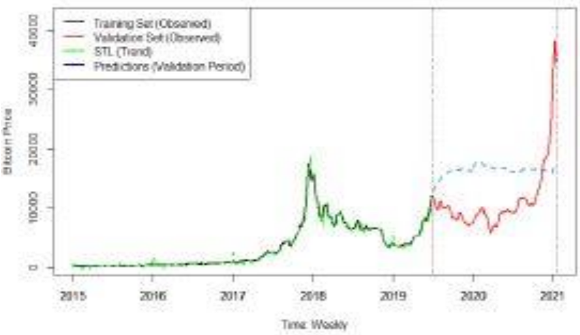

Figure 4g: Weekly predictions in the validation period (STL model)

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Monthly predictions in the validation period

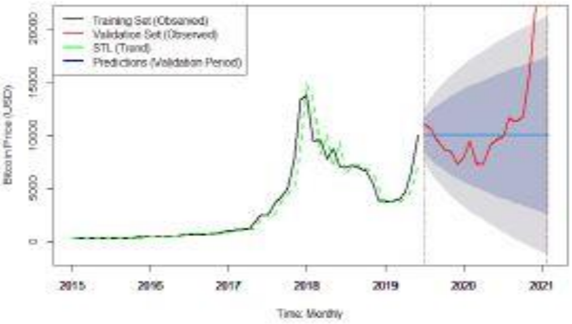

Figure 5a: Monthly predictions in the validation period (Naïve model)

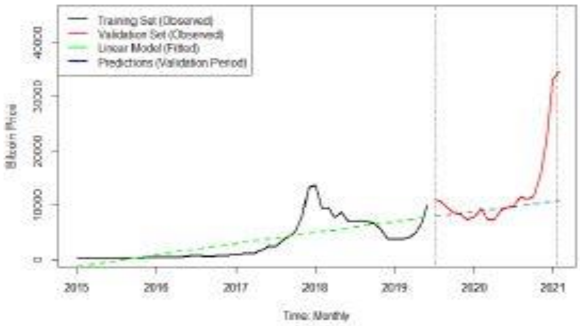

Figure 5b: Monthly predictions in the validation period (Linear model)

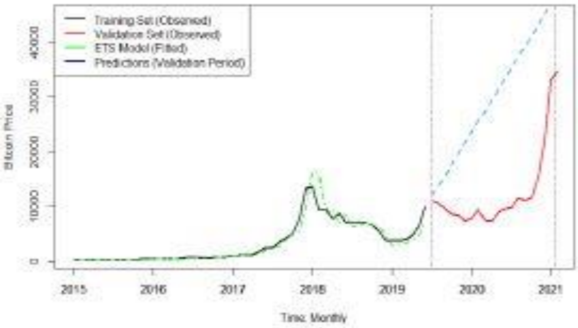

Figure 5c: Monthly predictions in the validation period (ETS model)

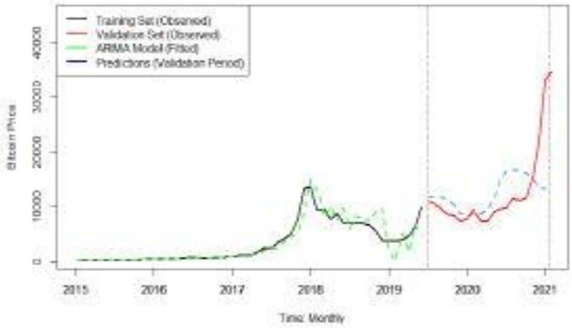

Figure 5d: Monthly predictions in the validation period (ARIMA model)

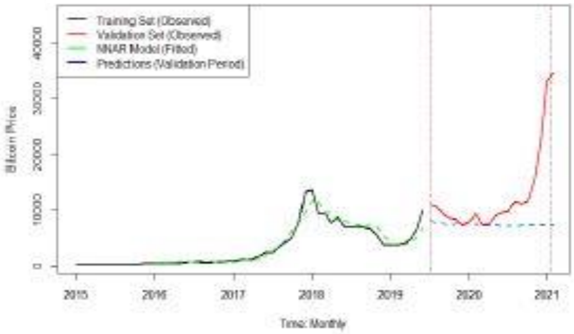

Figure 5e: Monthly predictions in the validation period (NNAR model)

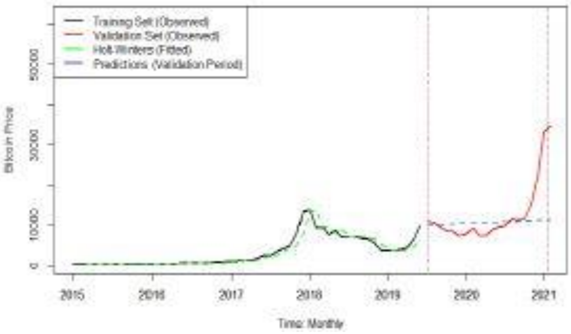

Figure 5f: Monthly predictions in the validation period (HWF model)

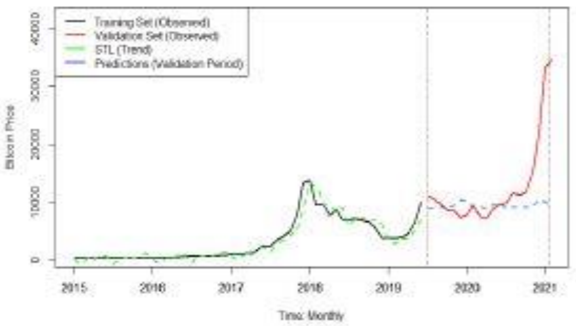

Figure 5g: Monthly predictions in the validation period (STL model)

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Table D.2: Autocorrelation and conditional heteroscedasticity tests

Forecast		Daily		Weekly		Monthly	
Methods	Test	Stat.	p-value	Stat.	p-value	Stat.	p-value
NAIVE	LBox (Q*)	885.20	(2.20×10^{-16})	101.71	(6.06×10^{-06})	8.2993	(0.686343)***
ETS	LBox (Q*)	552.19	(3.02×10^{-14})	54.494	(0.1124)***	4.9994	(0.623120)***
LINEAR	B-G (LM)	1634.3	(2.20×10^{-16})	228.13	(2.20×10^{-16})	44.965	(4.920×10^{-6})
ARIMA	LBox (Q*)	627.96	(2.20×10^{-16})	115.35	(6.99×10^{-8})	3.8399	(0.9543)***
NNAR	LBox (Q*)	513.31	(6.96×10^{-11})	63.276	(0.0185)	3.0353	(0.980623)
STL	B-G (LM)	1534.3	(2.30×10^{-15})	-	-	4.2532	(0.882145)
HWM	LBox(Q*)	563.27	(3.20×10^{-16})	87.255	(3.470×10^{-5})	51.483	(1.33×10^{-5})
NAIVE	QM (x^2)	0.6483	(0.9996)*	1.7075	(0.9887)	1.6734	(0.991271)
ETS	QM (x^2)	15.865	(0.0072)	39.657	(1.75×10^{-7})	17.745	(0.003284)
LINEAR	QM (x^2)	10.883	(0.0009707)	10.234	(39.657)	7.9473	(0.004816)
ARIMA	QM (x^2)	9.9624	(0.0016)	9.7093	(0.0018)	8.2571	(0.004059)
NNAR	QM (x^2)	15.674	$(7.53 \times 10^{-5*})$	15.161	(9.88×10^{-5})	9.7093	(0.001833)
STL	QM (x^2)	9.5214	(0.0023)	6.5773	(0.0103)	1.1091	(0.292334)
HWM	QM (x^2)	9.8933	(0.0017)	9.2731	(0.0023)	7.2250	(0.007190)

Breusch-Godfrey [B-G (LM)]; Ljung-Box (Q*) (x-square:), Qm (x^2) (p-values < 0.05) indicates the presence of conditional heteroscedasticity.
*** indicate no autocorrelation

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